EX. 1 :

He : GAZ MONOATOMIQUE $\rightarrow$ G.P.

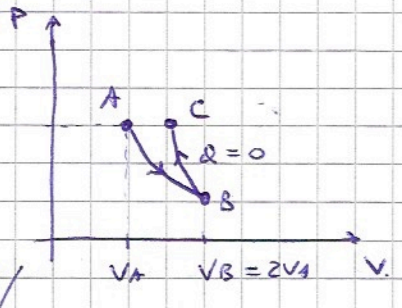
INITIALEMENT : T_A, V_A

T. QUASI-STATIQUE ISOTHERME : $V_A \rightarrow V_B = 2V_A$
[RÉVERSIBLE]

T. " " ADIABATIQUE : $P_B \rightarrow P_C = P_A$
[RÉVERSIBLE]

Q1) $c_V = \frac{3}{2} R \Rightarrow \gamma = \frac{c_P}{c_V} = \frac{c_V + R}{c_V} = 1 + \frac{R}{c_V} = 1 + \frac{\cancel{R}}{\frac{3}{2}\cancel{R}} = \frac{5}{3}$

Q2) ISOTH. :



$V_B = 2V_A$

$T_B = T_A$

$$\underbrace{P_B = \frac{m R T_B}{V_B}}_{\text{éq. état}} = \frac{m R T_A}{2V_A} = \frac{P_A}{2}$$

Q3) ADIAB. :

$P_C = P_A$

$PV^\gamma = \text{const} \Rightarrow P_C V_C^\gamma = P_B V_B^\gamma \Rightarrow V_C = V_B \left(\frac{P_B}{P_C}\right)^{\frac{1}{\gamma}} = 2V_A \left(\frac{1}{2}\right)^{3/5}$

$$\underbrace{T_C = \frac{P_C V_C}{m R}}_{\text{éq. état}} = \frac{P_A \, 2V_A (1/2)^{3/5}}{m R} = 2\left(\frac{1}{2}\right)^{3/5} T_A$$

Q4) $W = W_{AB} + W_{BC}$

$Q = Q_{AB} + \underbrace{Q_{BC}}_{0 \text{ (ADIAB.)}} = Q_{AB}$

ISOTH. : $\Delta U_{AB} = m c_V \Delta T_{AB} = 0 \quad (T_B = T_A) \Rightarrow$

$\Rightarrow Q_{AB} = -W_{AB}$; $W_{AB} = -\int_{V_A}^{V_B} p\, dV = -m R T_A \int_{V_A}^{V_B} \frac{1}{V} dV =$

$= -m R T_A \ln\left(\frac{V_B}{V_A}\right) = -m R T_A \ln 2$

ADIAB.: $W_{BC} = \Delta U_{BC} = n c_v (T_C - T_B) =$

$$= n c_v \left[2 T_A \left(\frac{1}{2}\right)^{3/5} - T_A \right] = n c_v T_A \left[2 \left(\frac{1}{2}\right)^{3/5} - 1 \right]$$

$$\Rightarrow W = W_{AB} + W_{BC} = - n R T_A \ln 2 - n c_v T_A \left[1 - 2 \left(\frac{1}{2}\right)^{3/5} \right] =$$

$$= - n R T_A \left\{ \ln 2 + \frac{3}{2} \left[1 - 2 \left(\frac{1}{2}\right)^{3/5} \right] \right\}$$

$c_v = \frac{3}{2} R$

$$Q = Q_{AB} = - W_{AB} = n R T_A \ln 2$$

Q5) $V_A = 1\,\ell$ $\qquad n = 4\,g$ $\qquad R \simeq 2077 \frac{J}{K\,kg}$

$T_A = 27\,°C \simeq 300\,K$

$$P_C = P_A = \frac{n R T_A}{V_A} \simeq \frac{4 \cdot 10^{-3}\, 2077\, (27 + 273)}{10^{-3}}\ Pa \simeq 25 \cdot 10^5\ Pa$$

$$T_C = 2 \left(\frac{1}{2}\right)^{3/5} T_A \simeq 396\,K \simeq 123\,°C$$

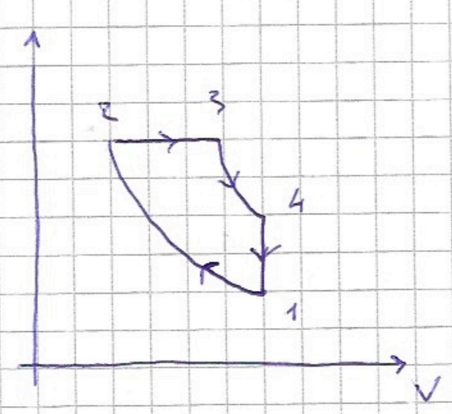
$$V_C = 2 \left(\frac{1}{2}\right)^{3/5} V_A \simeq 1{,}3 \cdot 10^{-3}\ m^3$$

$$W = - n R T_A \left\{ \ln 2 + \frac{3}{2} \left[1 - 2 \left(\frac{1}{2}\right)^{3/5} \right] \right\} \simeq -533\ J$$

$$Q = n R T_A \ln 2 \simeq 1728\ J$$

• EX. 2 :

Q1)



1 → 2 : ADIABATIQUE

2 → 3 : ISOBARE (COMBUSTION)

3 → 4 : ADIABATIQUE

4 → 1 : ISOCHORE

CYCLE MOTEUR ⇒ parcouru dans le sens des aiguilles de la montre.

Q2) $Q_{12} = 0$; $Q_{34} = 0$

$Q_{23} = \Delta H_{23}$; H : ENTHALPIE

$Q_{23} = Q_C > 0$ (reçue) ; $Q_{23} = m c_p (T_3 - T_2)$ [$p = $ const]

$Q_{41} = Q_F < 0$ (cédée) ; $Q_{41} = \Delta U_{41} = m c_v (T_1 - T_4)$ [$V = $ const]

Q3) 1° PRINCIPE : $\Delta U = W + Q_C + Q_F = 0$ (cycle) ⇒

$$\Rightarrow W = -Q_C - Q_F = -(Q_{23} + Q_{41})$$

$$W < 0 \text{ (MOTEUR)} \Rightarrow |Q_{41}| < Q_{23}$$

Q4) $$\eta = \frac{|W|}{Q_C} = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}}$$

à partir de Q2)

$Q_{23} = \Delta H_{23} = m c_p (T_3 - T_2)$ [$p = $ const] ⇒

$Q_{41} = \Delta U_{41} = m c_v (T_1 - T_4)$ [$V = $ const]

$$\Rightarrow \eta = 1 + \frac{c_v (T_1 - T_4)}{c_p (T_3 - T_2)}$$

Q5) LOI de LAPLACE : $PV^\gamma = $ const ⇒ $P_i V_i^\gamma = P_f V_f^\gamma$ ⇒

$$\Rightarrow \frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^\gamma \Rightarrow \frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma - 1}$$

en utilisant l'éq. d'état pour éliminer P_i, P_f

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} ; \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} \Rightarrow$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{\alpha}\right)^{\gamma-1} \qquad \alpha = \frac{V_1}{V_2}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{V_4}{V_2}\,\frac{V_2}{V_3}\right)^{\gamma-1} = \qquad \beta = \frac{V_3}{V_2}$$

$$= \left(\frac{V_1}{V_2}\,\frac{V_2}{V_3}\right)^{\gamma-1} = \left(\frac{\alpha}{\beta}\right)^{\gamma-1}$$

$V_4 = V_1$ (ISOCHORE)

Q6) $2 \to 3$: ISOBARE

$$P_2 = P_3 \iff \frac{nR\,T_2}{V_2} = \frac{nR\,T_3}{V_3} \Rightarrow T_3 = T_2\,\frac{V_3}{V_2} = T_2\,\beta$$

Q7) $$\eta = 1 + \underbrace{\frac{C_v}{C_p}}_{=\frac{1}{\gamma}}\,\frac{T_1 - T_4}{T_3 - T_2} = 1 + \frac{1}{\gamma}\,\frac{T_2\,\alpha^{1-\gamma} - T_3\,(\alpha/\beta)^{1-\gamma}}{T_3 - T_2} =$$

$$T_1 = T_2\,\alpha^{1-\gamma}$$

$$T_4 = T_3\left(\frac{\alpha}{\beta}\right)^{1-\gamma}$$

$$T_3 = \beta\,T_2$$

$$= 1 + \frac{1}{\gamma}\,\frac{\cancel{T_2}\left[\alpha^{1-\gamma} - \beta\,(\alpha/\beta)^{1-\gamma}\right]}{\cancel{T_2}\,[\beta - 1]} =$$

$$= 1 + \frac{1}{\gamma}\,\alpha^{1-\gamma}\,\frac{\left[1 - \beta^{\gamma}\right]}{\beta - 1} \Rightarrow$$

$$\Rightarrow \quad \eta = 1 - \frac{1}{\gamma\,\alpha^{\gamma-1}}\,\frac{1-\beta^{\gamma}}{1-\beta}$$

Q8) $\gamma = 1{,}4$; $\alpha = 16$; $\beta = 1{,}5$ $\Rightarrow$ $\eta \simeq 0{,}64$