• EX. 1

Q1) $\vec{\nabla}\cdot\vec{u} = \frac{\partial u}{\partial x} + \underbrace{\cancel{\frac{\partial v}{\partial y}}}_{=0} = \frac{\partial u}{\partial x} = 0$ car $u = u(y)$; $\vec{u} = (u, v)$

Q2) $u(y=0) = 0$ à cause du frottement visqueux à la paroi.

Q3) $u(y^*) = u^*$; $\left.\frac{du}{dy}\right|_{y=y^*} = 0$; $u(y) = a\,y^2 + by + c$

$u(y=0) = 0 \Rightarrow c = 0$

$0 = \left.\frac{du}{dy}\right|_{y=y^*} = 2ay^* + b \Rightarrow b = -2ay^*$

$u(y) = a\,y\,(y - 2y^*)$

$u^* = u(y^*) = a\,y^{*2} - 2a\,y^{*2} = -a\,y^{*2} \Rightarrow a = -\frac{u^*}{y^{*2}}$

$\Rightarrow \boxed{u(y) = -\frac{u^*}{y^{*2}}\,y\,(y - 2y^*)}$

Q4) $\tau = \eta\frac{du}{dy} = -2\eta\frac{u^*}{y^{*2}}(y - y^*)$

F. NEWTON

Q5) $\tau = 0 \Rightarrow y = y^*$ (où $u(y)$ est max)

$|\tau| = 2\eta\frac{u^*}{y^{*2}}|y - y^*| \underset{y \leq y^*}{=} 2\eta\frac{u^*}{y^{*2}}(y^* - y)$

$|\tau| \rightarrow$ max pour $y = 0$ (à la paroi)

$|\tau|_{max} = 2\eta\frac{u^*}{y^*}$

• EX. 2

Q1) $Re_L = \frac{VL}{\nu} = 3 \cdot 10^8$

$Re_L > 10^7 \Rightarrow$ C.L. TURBULENTE

Q2) $Re_L > 10^7 \Rightarrow C_F = 0{,}455 \left(\log_{10} Re_L\right)^{-2{,}58} \simeq 0{,}0018$

$F = C_F \, \rho \frac{V^2}{2} S \simeq 16{,}8 \cdot 10^3 \ N$

Q3) $W = FV \simeq 75{,}6 \cdot 10^3 \ W$

• EX. 3

Q1) $V = \frac{4}{\pi} \frac{q_v}{d^2} \simeq 3{,}18 \ m/s$

$q_v = V \cdot S \ ; \ S = \frac{\pi d^2}{4}$

Q2) $Re = \rho \frac{V d}{\eta} = 31800$

$\underbrace{2000}_{Re_{c1}} < Re < \underbrace{10^5}_{Re_{c2}} \Rightarrow$ écoulement turbulent lisse

Re c1: transition laminaire turbulent

Re c2: transition lisse rugueux

Q3) $2000 < Re < 10^5 \rightarrow$ F. BLASIUS: $\boxed{\Lambda = 0{,}316 \ Re^{-1/4} \simeq 0{,}024}$

F. POISEUILLE : régime laminaire

F. BLASIUS : " turbulent lisse

F. BLENCH : " " rugueux

P.C. LIN. TOT. (p.u. de masse) : $J_L = \Lambda \frac{V^2}{2} \frac{L_{TOT}}{d} \simeq 145{,}62 \ \frac{J}{Kg} = 145{,}62 \ \frac{m^2}{s^2}$

$L_{TOT} = n_L \cdot L = 12 \cdot 1 \ m = 12 \ m$

$n_L = 12$ tubes rectilignes

$L = 1 \ m$

Q4) $J_s = n_s K_s \frac{v^2}{2} \simeq 22{,}25 \ \frac{m^2}{s^2} = 22{,}25 \ \frac{J}{kg}$

$n_s = 11$ coudes à $180°$

$K_s = 0{,}4$

Q5) TH. BERNOULLI GÉNÉRALISÉ entre (1) et (2) :

$$\cancel{\frac{\rho V_1^2}{2}} + \cancel{\rho g z_1} + p_1 = \cancel{\frac{\rho V_2^2}{2}} + \cancel{\rho g z_2} + p_2 + \rho (J_L + J_s)$$

$V_1 = V_2$

$z_1 = z_2$ $\Rightarrow$

$\Rightarrow p_2 = p_1 - \rho (J_L + J_s) = 3 \cdot 10^5 \ Pa - 10^3 \ \frac{kg}{m^3} \left(145{,}62 + 22{,}25\right) \frac{m^2}{s^2} \simeq$

$\simeq 1{,}32 \cdot 10^5 \ Pa = 1{,}32$ bar