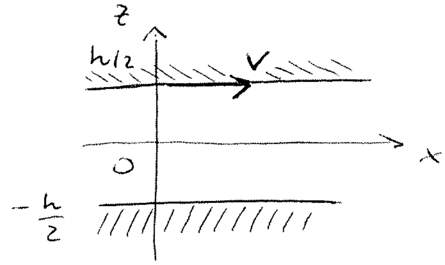
EX. 1 :



$$\mu \frac{\partial^2 v}{\partial z^2} = \frac{\partial p}{\partial x}$$

$$v = v(z) \Rightarrow \frac{\partial v}{\partial z} = \frac{dv}{dz}$$

$$p = p(x) \Rightarrow \frac{\partial p}{\partial x} = \frac{dp}{dx} \quad ; \quad \frac{dp}{dx} = G$$

Q1)

$$\frac{d^2 v}{dz^2} = \frac{G}{\mu} \Rightarrow \frac{dv}{dz} = \frac{G}{\mu} z + A$$

$$v(z) = \frac{G}{2\mu} z^2 + Az + B$$

Q2)

$$\begin{cases} v\left(-\frac{h}{2}\right) = 0 \\ v\left(\frac{h}{2}\right) = V \end{cases} \Rightarrow \begin{cases} 0 = \frac{G}{2\mu}\frac{h^2}{4} - \frac{Ah}{2} + B & (i) \\ V = \frac{G}{2\mu}\frac{h^2}{4} + \frac{Ah}{2} + B & (ii) \end{cases}$$

$$(i)+(ii) \Rightarrow V = \frac{G}{2\mu}\frac{h^2}{2} + 2B \Rightarrow B = \left(V - \frac{Gh^2}{4\mu}\right)\frac{1}{2} \Rightarrow$$

$$(ii)-(i) \Rightarrow V = Ah \Rightarrow A = \frac{V}{h}$$

$$\Rightarrow v(z) = \frac{G}{2\mu} z^2 + \frac{V}{h} z + \frac{V}{2} - \frac{Gh^2}{8\mu} =$$

$$= \frac{G}{8\mu}\left(4z^2 - h^2\right) + \frac{V}{h} z + \frac{V}{2}$$

Q3)

$$Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} v(z)\,dz = \frac{G}{2\mu}\frac{z^3}{3}\Bigg|_{-\frac{h}{2}}^{\frac{h}{2}} - \frac{Gh^2}{8\mu} z\Bigg|_{-\frac{h}{2}}^{\frac{h}{2}} + \frac{V}{h}\frac{z^2}{2}\Bigg|_{-\frac{h}{2}}^{\frac{h}{2}} + \frac{V}{2} h =$$

$$= \frac{Vh}{2} - \frac{Gh^3}{12\mu}$$

Q4) $u(z) = \frac{G}{8\mu}\left(4z^2 - h^2\right) + \frac{V}{h} z + \frac{V}{2}$

$G = 0 \Rightarrow U(z) = \frac{V}{h} z + \frac{V}{2}$ PROFIL LINÉAIRE

$\left(\frac{dp}{dx} = 0\right)$ ECOUL. CISAILLEMENT SIMPLE

$V = 0 \Rightarrow u(z) = \frac{G}{8\mu}\left(4z^2 - h^2\right)$ PROFIL PARABOLIQUE

ECOUL. de POISEUILLE

$G < 0 \Rightarrow \frac{dp}{dx} < 0 \Rightarrow p \searrow$ avec $x \nearrow$ $\Rightarrow$ le gradient de pression est "moteur" et amplifie l'effet produit par la plaque en $z = \frac{h}{2}$.

$G > 0 \Rightarrow \frac{dp}{dx} > 0 \Rightarrow p \nearrow$ avec $x \nearrow$ $\Rightarrow$ le gradient de pression est dit ADVERSE et s'oppose à l'action de la plaque.

Q5) $\tau = \mu \frac{\partial u}{\partial z} = \mu\left(\frac{G}{\mu} z + \frac{V}{h}\right) = G z + \frac{V\mu}{h}$

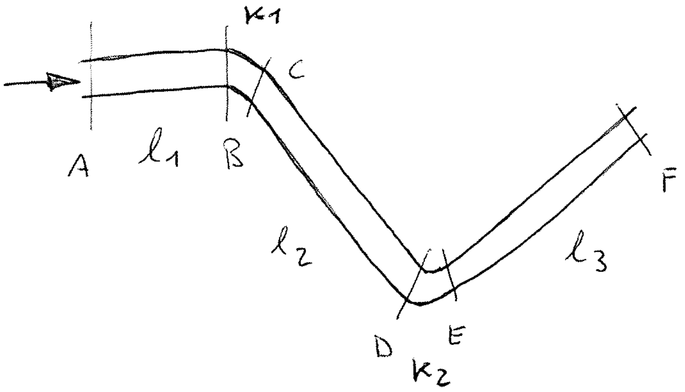
F. NEWTON

$\tau\left(z = \frac{h}{2}\right) = \frac{Gh}{2} + \frac{V\mu}{h}$

$\tau(z^*) = 0 \Rightarrow z^* = -\frac{V\mu}{Gh} < 0$

pas au centre du système ($z=0$) comme pour l'écoulement de Poiseuille.

EX. 2 :



$d = 100$ mm

$l_1 = 6$ m

$l_2 = 12$ m

$l_3 = 5$ m

$K_1 = 0,2$ (P.C. SING.)

$K_2 = 0,3$ (P.C. SING.)

$P_A = 8$ bar

$q_v = 2,5$ l/s

$\rho = 900$ kg/m³

$\mu = 0,7$ Pa.s

Q1) $V = \dfrac{4 q_v}{\pi d^2} \simeq 0,318 \ \dfrac{m}{s}$

Q2) $Re = \dfrac{V d}{\nu} \simeq 40,8$

$\nu = \dfrac{\mu}{\rho} \simeq 7,8 \cdot 10^{-4} \ m^2 s^{-1}$

Q3) $Re < Re_c = 2000 \Rightarrow$ ECOULEMENT LAMINAIRE

Q4) $\Lambda = \dfrac{64}{Re} \simeq 1,57$

Q5) $p_A + \cancel{\rho g z_A} + \cancel{\rho \frac{V_A^2}{2}} = p_F + \cancel{\rho g z_F} + \cancel{\rho \frac{V_F^2}{2}} + \xi_{LIN} + \xi_{SING}$

$z_A = z_F$ (HORIZ.) ; $V_A = V_F$

$\xi_{LIN} = \xi_{LIN}^{(l_1)} + \xi_{LIN}^{(l_2)} + \xi_{LIN}^{(l_3)}$

$\xi_{SING} = \xi_{SING}^{(k_1)} + \xi_{SING}^{(k_2)}$

$\Rightarrow p_A = p_F + \xi_{LIN} + \xi_{SING}$

$\xi_{LIN}^{(l_i)} = \frac{\lambda \rho V^2}{2} \frac{l_i}{d}$

Q6) $\xi_{LIN} = \xi_{LIN}^{(l_1)} + \xi_{LIN}^{(l_2)} + \xi_{LIN}^{(l_3)} =$

$= \frac{\lambda \rho V^2}{2d} (l_1 + l_2 + l_3) = \frac{1{,}57 \cdot 900}{2 \cdot 0{,}1} 0{,}318^2 (6 + 12 + 5) \simeq$

$\simeq 16432$ Pa

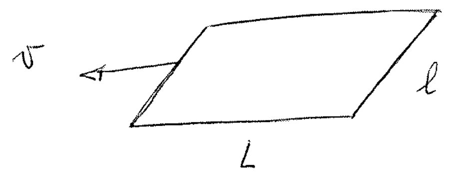
Q7) $\xi_{SING} = k_1 \frac{\rho V^2}{2} + k_2 \frac{\rho V^2}{2} = \frac{\rho V^2}{2} (k_1 + k_2) = \frac{900 \cdot 0{,}318^2}{2} (0{,}2 + 0{,}3) \simeq$

$\simeq 23$ Pa

Q8) $p_F = p_A - \xi_{LIN} - \xi_{SING} = 8 \cdot 10^5 - 16432 - 23 \simeq 783545$ Pa $\simeq 7{,}8$ bar

$1 \text{ bar} = 10^5$ Pa

x.3 :



$v = 0{,}9 \ m\,s^{-1}$

$\mu = 10^{-3} \ Pa \cdot s$

$L = 6 \ m$

$\ell = 1 \ m$

$\rho = 10^{3} \ kg/m^{3}$

$R_c = 5 \cdot 10^{5}$

Q1) $R_x = \dfrac{x v}{\nu}$

$R_x = R_c \Rightarrow x^* = \dfrac{R_c \nu}{v} \simeq 0{,}55 \ m$

$\nu = \dfrac{\mu}{\rho} = 10^{-6} \ \dfrac{m^2}{s}$

$\dfrac{x^*}{L} = \dfrac{0{,}55}{6} \simeq 0{,}092 < 0{,}1$

Q2) $\dfrac{\delta}{x} \sqrt{R_x} = 5$

$\begin{matrix} x = x^* \\ R_x = R_c \end{matrix} \Rightarrow \delta = \dfrac{5 x^*}{\sqrt{R_c}} \simeq 0{,}0039 \ m = 3{,}9 \ mm$

Q3) $R_L = \dfrac{v L}{\nu} = \dfrac{0{,}9 \cdot 6}{10^{-6}} \simeq 5{,}4 \cdot 10^{6}$

Q4) $R_L < 10^{7} \Rightarrow C_x = \dfrac{0{,}074}{R_L^{1/5}} \simeq 0{,}003332$

Q5) $F = C_x \, \rho \dfrac{S v^2}{2} \simeq 8{,}1 \ N$

$\uparrow$

$S = L\ell = 6 \ m^2$

Q6) $T = 2F \simeq 16{,}2 \ N$