

Q1) EQ. NAVIÉ-STOKES :  $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \vec{u} + \vec{g}$   
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 ACCÉLÉRATION de la PARTICULE

TERME NON LINÉAIRE (INÉRTIEL) :  $\vec{u} \cdot \nabla \vec{u}$

$Re \approx \frac{T. INÉRTIEL}{T. VISQUEUX}$  ;  $Re \ll 1 \Rightarrow \frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho} \nabla p + \nu \Delta \vec{u} + \vec{g}$  EQ. STOKES

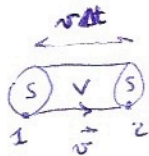
$Fr \approx \frac{EN. CINÉTIQUE}{EN. POTENTIELLE (g)}$  ;  $Fr \gg 1 \Rightarrow \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \vec{u}$

Q2) PUISSANCE DISSIPÉE par les PERTES de CHARGE :

$W = \sum_{12} qv$   
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 P.C.      DÉBIT

$qv = v \cdot S = \frac{\Delta V}{\Delta t}$

DÉBIT VOLUMÉTRIQUE



TH. BERNOULLI généralisée :  $\left( \frac{\rho v_1^2}{2} + p_1 + \rho g z_1 \right) - \left( \frac{\rho v_2^2}{2} + p_2 + \rho g z_2 \right) = \sum_{12}$   
 énergie p. unité de volume →  $\frac{E_1}{\Delta V}$  →  $\frac{E_2}{\Delta V}$

$\Rightarrow \left( qv \left( \frac{E_1}{\Delta V} - \frac{E_2}{\Delta V} \right) \right) = \sum_{12} \cdot qv \Rightarrow \frac{\Delta E}{\Delta t} = \sum_{12} \cdot qv \Rightarrow W = \sum_{12} \cdot qv$   
 avec  $\Delta E = E_1 - E_2$  car  $W = \frac{\Delta E}{\Delta t}$

Q3) CONDUITE HORIZONTALE en RÉGIME PERMANENT ÉTABLI

Loi de COLEBROOK-WHITE

$\frac{1}{\sqrt{\lambda}} = -2 \log_{10} \left( \frac{2,51}{Re \sqrt{\lambda}} + \frac{\epsilon}{3,71D} \right)$   
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 COEFF. P.C. UNITAIRE          LISSE                  RUGUEUX

$\mu, \rho, L, \epsilon$  : CONNUS

$D, qv$  CONNUS  $\Rightarrow \left( \frac{E}{D} \right)$  CONNU ;  $U = \frac{qv}{\frac{\pi D^2}{4}}$  et  $Re = \frac{UD}{\nu}$  CONNU  $\Rightarrow$  LECTURE DIRECTE dans le GRAPHIQUE :  $\lambda$

$\lambda_{MORSE} = \frac{\lambda L}{D} \rho \frac{U^2}{2}$  P.C.

$$\vec{v} = (u, v); u(y) = \frac{1}{4} y^2; v = 0$$

$$Q1) \quad \vec{\nabla} \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \text{INCOMPRESSIBLE}$$

$$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$$

$$Q2) \quad [u] = [L t^{-1}]$$

$$[y^2] = [L^2] \Rightarrow [a] = [L^{-1} t^{-1}] \rightarrow m^{-1} s^{-1}$$

$$Q3) \quad \tau = \mu \frac{du}{dy} = \mu \frac{z}{4} y = \frac{\mu}{2} y$$

$$Q4) \quad u \sim y^2; \tau \sim y \quad \text{comme pour l'écoulement de POISEUILLE.}$$

$$Q5) \quad [\tau] = \left[ \frac{F}{L^2} \right] \rightarrow N m^{-2}$$

$$Q6) \quad \tau_p = \tau(y=0) = 0 \quad N m^{-2}$$

$$y^* = 3 \text{ cm}; \quad \tau(y^*) = \frac{\mu}{2} y^* = \frac{\rho \nu}{2} y^* = \frac{3 \cdot 10^{-2} \text{ m} \cdot 10^3 \frac{\text{kg}}{\text{m}^3} \cdot 10^{-6} \frac{\text{m}^2}{\text{s}}}{2 \text{ m s}} =$$

$$\nu = 10^{-6} \text{ m}^2 \text{ s}^{-1}$$

$$\rho = 10^3 \text{ kg m}^{-3}$$

$$y^* = 3 \cdot 10^{-2} \text{ m}$$

$$= 1,5 \cdot 10^{-5} \frac{\text{kg m s}^{-2}}{\text{m}^2} = 1,5 \cdot 10^{-5} \frac{\text{N}}{\text{m}^2}$$

$$Q7) \quad \Delta_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\Delta = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 & \frac{y}{4} \\ \frac{y}{4} & 0 \end{pmatrix}$$

$$Q8) \quad \text{tr}(\Delta) = \sum_{i=1}^2 \Delta_{ii} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{car} \quad \vec{\nabla} \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

et parce que  $u = u(y)$  et  $v = 0$  (cisaillement)  $\Rightarrow$

$$\Rightarrow \frac{\partial u}{\partial x} = 0 \quad \text{et} \quad \frac{\partial v}{\partial y} = 0 \quad \text{aussi séparément.}$$

Q1) EQ. BERNOULLI entre A et S:

$$P_A + \frac{\rho}{2} v_A^2 + \rho g z_A = P_S + \frac{\rho v_S^2}{2} + \rho g z_S + \underbrace{\sum_{LIN}}_{\frac{\lambda(L_{AM}+L_{MS})}{D}} + \underbrace{\sum_{SING}}_{\frac{k\rho v_S^2}{2}}$$

.. Q2)  $v_A \ll v_S$  (réservoir)  
 $P_A = P_S = P_{atm}$   $\Rightarrow$

$\Rightarrow$  EQ. BERNOULLI entre A et S:

$$\rho g \underbrace{(z_A - z_S)}_H = \frac{\rho}{2} v_S^2 + \frac{\lambda}{D} (L_{AM} + L_{MS}) \frac{\rho}{2} v_S^2 + \frac{k\rho}{2} v_S^2 \Rightarrow$$

$$\Rightarrow \frac{\rho}{2} \left[ 1 + \frac{\lambda}{D} (L_{AM} + L_{MS}) + k \right] v_S^2 = \rho g H \Rightarrow$$

$$\Rightarrow v_S = \sqrt{\frac{2gH}{1 + \frac{\lambda}{D} (L_{AM} + L_{MS}) + k}}$$

$$; v_S \approx 4,97 \text{ m/s}$$

Q3) EQ. BERNOULLI entre A et M:

$$P_A + \frac{\rho}{2} v_A^2 + \rho g z_A = P_M + \frac{\rho}{2} v_M^2 + \rho g z_M + \sum_{LIN}; \sum_{SING} \text{ est NÉGATIVE}$$

↑  
car réservoir

Q4)  $v_M = v_S$

$v_A \ll v_S$  (réservoir)

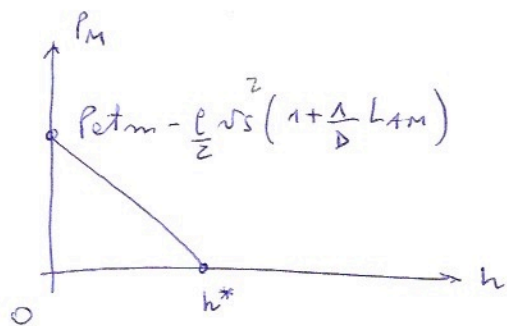
$$\Rightarrow P_M = P_{atm} + \rho g \underbrace{(z_A - z_M)}_{-h} - \frac{\rho v_S^2}{2} - \frac{\lambda L_{AM}}{D} \frac{\rho v_S^2}{2}$$

$$P_A = P_{atm}$$

$$\sum_{LIN} = \frac{\lambda}{D} L_{AM} \frac{\rho}{2} v_S^2$$

$$\Rightarrow P_M = P_{atm} - \rho g h - \frac{\rho}{2} v_S^2 \left[ 1 + \frac{\lambda}{D} L_{AM} \right]$$

Q5).



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Q6)

$$h^* : p_m = 0 \Rightarrow$$

$$h^* = \frac{1}{\rho g} \left[ p_{atm} - \frac{\rho}{2} v_s^2 \left(1 + \frac{A}{A_m}\right) \right]$$

$$h^* \approx 8,03 \text{ m}$$

