PARTIE 1

Q1) EQ. NAVIE-STOKES : $\frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \nu\Delta\vec{v} + \vec{g}$

ACCÉLÉRATION de la PESANTEUR

TERME NON LINÉAIRE (INERTIEL) : $\vec{v}.\vec{\nabla}\vec{v}$

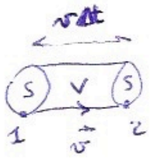
$Re \approx \frac{\text{T. INERTIEL}}{\text{T. VISQUEUX}}$; $Re \ll 1 \Rightarrow \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\vec{\nabla}p + \nu\Delta\vec{v} + \vec{g}$ EQ. STOKES

$Fr \approx \frac{\text{EN. CINÉTIQUE}}{\text{EN. POTENTIELLE } (g)}$; $Fr \gg 1 \Rightarrow \frac{\partial \vec{v}}{\partial t} + \vec{v}.\vec{\nabla}\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \nu\Delta\vec{v}$

Q2) PUISSANCE DISSIPÉE par les PERTES de CHARGE : $W = \xi_{12}\, q_v$

P.C. DÉBIT

$q_v = v.S = \frac{\Delta V}{\Delta t}$ DÉBIT VOLUMÉTRIQUE



TH. BERNOULLI généralisé : $\underbrace{\left(\frac{\rho v_1^2}{2} + p_1 + \rho g z_1\right)}_{\frac{E_1}{\Delta V}} - \underbrace{\left(\frac{\rho v_2^2}{2} + p_2 + \rho g z_2\right)}_{\frac{E_2}{\Delta V}} = \xi_{12}$

énergie p. unité de volume

$\Rightarrow \underbrace{q_v}_{=\frac{\Delta V}{\Delta t}}\left(\frac{E_1}{\Delta V} - \frac{E_2}{\Delta V}\right) = \xi_{12}\cdot q_v \Rightarrow \frac{\Delta E}{\Delta t} = \xi_{12}\cdot q_v \Rightarrow W = \xi_{12}\cdot q_v$

avec $\Delta E = E_1 - E_2$

car $W = \frac{\Delta E}{\Delta t}$

Q3) CONDUITE HORIZONTALE en RÉGIME PERMANENT ÉTABLI

$$\frac{1}{\sqrt{\Lambda}} = -2\log_{10}\left(\frac{2{,}51}{Re\sqrt{\Lambda}} + \frac{\varepsilon}{3{,}71D}\right)$$

LOI de COLEBROOK-WHITE

COEFF. P.C. UNITAIRE

LISSE RUGUEUX

$\mu, \rho, L, \varepsilon$: CONNUS

D, q_v CONNUS $\Rightarrow \frac{\varepsilon}{D}$ CONNU ; $U = \frac{q_v}{\frac{\pi D^2}{4}}$ et $Re = \frac{UD}{\nu}$ CONNU $\Rightarrow$ LECTURE DIRECTE dans le GRAPHIQUE : Λ

$\nu = \mu/\rho$

$\boxed{\xi_{LIN} = \frac{\Lambda L}{D}\rho\frac{U^2}{2}}$ P.C.

PARTIE 2

. EX. 1

$\vec{U} = (u, v)$; $u(y) = \frac{1}{4} y^2$; $v = 0$

Q1) $\vec{\nabla} \cdot \vec{U} = \underset{\overset{\shortparallel}{0}}{\frac{\partial u}{\partial x}} + \underset{\overset{\shortparallel}{0}}{\frac{\partial v}{\partial y}} = 0 \Rightarrow$ INCOMPRESSIBLE

Q2) $[u] = [L t^{-1}]$

$[y^2] = [L^2] \Rightarrow [a] = [L^{-1} t^{-1}] \rightarrow m^{-1} s^{-1}$

Q3) $\tau = \mu \frac{du}{dy} = \mu \frac{\cancel{2}^{1}}{\cancel{4}_{2}} y = \frac{\mu}{2} y$

Q4) $u \sim y^2$; $\tau \sim y$ comme pour l'écoulement de POISEUILLE.

Q5) $[\tau] = \left[\frac{F}{L^2}\right] \rightarrow N m^{-2}$

Q6) $\tau_p = \tau(y=0) = 0 \ N m^{-2}$

$y^* = 3\,cm$; $\tau(y^*) = \frac{\mu}{2} y^* = \frac{\rho \nu}{2} y^* = \frac{3 \cdot 10^{-2}\, m}{2\ m\ s} \cdot 10^3 \frac{kg}{\cancel{m^3}} \cdot 10^{-6} \frac{\cancel{m^2}}{s} =$

$\nu = 10^{-6} m^2 s^{-1}$

$\rho = 10^3 \,kg\, m^{-3}$

$y^* = 3 \cdot 10^{-2} m$

$= 1{,}5 \cdot 10^{-5} \frac{kg\ m\ s^{-2}}{m^2} = 1{,}5 \cdot 10^{-5} \frac{N}{m^2}$

Q7) $D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

$$D = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 & \frac{y}{4} \\ \frac{y}{4} & 0 \end{pmatrix}$$

Q8) $tr(D) = \sum_{i=1}^{2} D_{ii} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ car $\vec{\nabla} \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

et parce que $u = u(y)$ et $v = 0$ (cisaillement) $\Rightarrow$

$\Rightarrow \frac{\partial u}{\partial x} = 0$ et $\frac{\partial v}{\partial y} = 0$ aussi séparément.

EX.2

Q1) EQ. BERNOULLI entre A et S:

$$p_A + \frac{\rho}{2} v_A^2 + \rho g z_A = p_S + \frac{\rho v_S^2}{2} + \rho g z_S + \underbrace{\xi_{LIN}}_{= \frac{\lambda (L_{AM}+L_{MS})}{D} \frac{\rho}{2} v_S^2} + \xi_{SING} \quad = \frac{k\rho}{2} v_S^2$$

Q2) $v_A << v_S$ (RÉSERVOIR) $\Rightarrow$

$p_A = p_S = p_{atm}$

$\Rightarrow$ EQ. BERNOULLI entre A et S:

$$\rho g \underbrace{(z_A - z_S)}_{H} = \frac{\rho}{2} v_S^2 + \frac{\lambda}{D} (L_{AM} + L_{MS}) \frac{\rho}{2} v_S^2 + \frac{k\rho}{2} v_S^2 \Rightarrow$$

$$\Rightarrow \frac{\rho}{2} \left[1 + \frac{\lambda}{D} (L_{AM} + L_{MS}) + k \right] v_S^2 = \rho g H \Rightarrow$$

$$\Rightarrow \boxed{v_S = \sqrt{\frac{2 g H}{1 + \frac{\lambda}{D} (L_{AM} + L_{MS}) + k}}} \quad ; \quad v_S \simeq 4{,}97 \text{ m/s}$$

Q3) EQ. BERNOULLI entre A et M:

$$p_A + \cancel{\frac{\rho}{2} v_A^2} + \rho g z_A = p_M + \frac{\rho}{2} v_M^2 + \rho g z_M + \xi_{LIN} \; ; \; \xi_{SING} \text{ est } \underline{\text{NÉGLIGÉ}} \text{ ici}$$

$\uparrow$

car réservoir

Q4) $v_M = v_S$

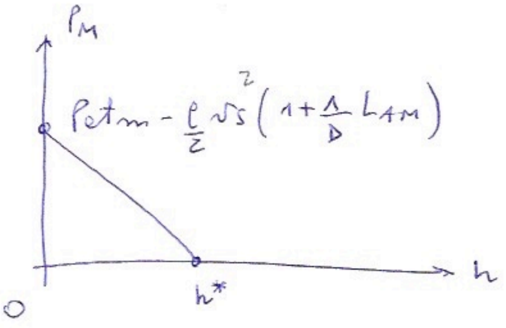
$v_A << v_S$ (réservoir) $\Rightarrow p_M = p_{atm} + \rho g \underbrace{(z_A - z_M)}_{-h} - \frac{\rho v_S^2}{2} - \frac{\lambda}{D} L_{AM} \frac{\rho}{2} v_S^2$

$p_A = p_{atm}$

$\xi_{LIN} = \frac{\lambda}{D} L_{AM} \frac{\rho}{2} v_S^2$

$$\Rightarrow \boxed{p_M = p_{atm} - \rho g h - \frac{\rho}{2} v_S^2 \left[1 + \frac{\lambda}{D} L_{AM} \right]}$$

Q5) :



Q6) $h^* : p_M = 0 \Rightarrow$ $\boxed{h^* = \frac{1}{\rho g}\left[p_{atm} - \frac{\rho}{2} v_s^2 \left(1 + \frac{\lambda}{D} L_{AM}\right)\right]}$

$h^* \simeq 8{,}03\ m$

- EX. 3

$$\frac{u}{U} = \sin\left(\frac{\pi y}{2\delta}\right)$$

C.L. LAMINAIRE

$U = 15\ m\,s^{-1}$

$L = 2\,m$

$\nu = 1{,}5 \cdot 10^{-5}\ m^2 s^{-1}$

$\rho = 1{,}2\ kg\,m^{-3}$

Q1) $R_L = \frac{UL}{\nu} = 2 \cdot 10^6$

- Q2) $\tau = \mu \frac{du}{dy} = \frac{\mu U \pi}{2\delta} \cos\left(\frac{\pi y}{2\delta}\right)$

$$\tau_p = \tau(y=0) = \frac{\mu U \pi}{2\delta} = \frac{\nu \rho \pi U}{2\delta}$$

Q3) $R_c = 10^5$

POINT de TRANSITION x^* : $R_{x^*} = R_c \iff \frac{U x^*}{\nu} = R_c \Rightarrow x^* = R_c \frac{\nu}{U} = 0{,}1\,m$

- Q4) $\delta^* = \delta(x^*) = 4{,}789 \frac{x^*}{\sqrt{R_{x^*}}} \simeq 0{,}0015\,m = 1{,}5\,mm$

$$\tau_p^* = \tau_p(\delta^*) = \frac{\nu\rho\pi U}{2\delta^*} \simeq 0{,}28\ \frac{N}{m^2}$$

- Q5) $C_F = \frac{1}{x^*}\int_0^{x^*} cf(x)\,dx$ COEFF. FROTTEMENT MOYEN pour $0 \leq x \leq x^*$ (C.L. LAMINAIRE)

$cf(x) = \frac{0{,}656}{\sqrt{R_x}}$ COEFF. FROTTEMENT LOCAL

$$C_F = \frac{1}{x^*}\int_0^{x^*} cf(x)\,dx = \frac{0{,}656}{x^*}\sqrt{\frac{\nu}{U}}\int_0^{x^*} x^{-1/2}\,dx =$$

$$= \frac{0{,}656}{x^*}\sqrt{\frac{\nu}{U}}\, 2x^{1/2}\Big|_0^{x^*} = \frac{2 \cdot 0{,}656}{\sqrt{\frac{U x^*}{\nu}}} = \frac{1{,}312}{\sqrt{R_{x^*}}} \simeq 0{,}004$$

Q6) $F = C_F\, \rho\, S \frac{V^2}{2}$ FORCE de FROTTEMENT (TRAÎNÉE)

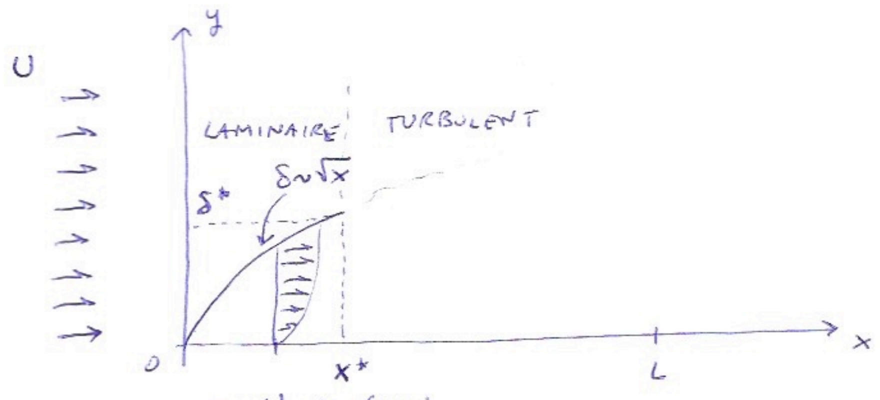
$S = x^* \cdot \ell$

$x^* = 0{,}1\,m\ ;\ \ell = 4\,m \Rightarrow S = 0{,}4\,m^2 \Rightarrow F \simeq 0{,}004 \cdot 1{,}2 \cdot 0{,}4 \cdot \frac{15^2}{2} = 0{,}216\,N$

$V = U = 15\,m\,s^{-1}$

$$\overline{\tau_p} = \frac{F}{S} \simeq 0{,}54\ \frac{N}{m^2}$$

Q7)



$u = U \sin\left(\frac{\pi y}{2\delta}\right)$

$y = 0 \Rightarrow u = 0$

$y = \delta \Rightarrow u = U \sin\left(\frac{\pi}{2}\right) = U$

$0 < y < \delta \Rightarrow u(y) \uparrow$ avec $y \uparrow$

$$\delta = 4{,}789 \frac{x}{\sqrt{R_x}} \sim \frac{x}{\sqrt{\frac{Ux}{\nu}}} \sim \sqrt{x}$$