

Introduction à la turbulence

Stefano Berti - Unité de Mécanique de Lille
stefano.berti@polytech-lille.fr
www.bertistefano.eu

Un phénomène très répandu



Tache rouge de Jupiter [NASA/JPL/Space Science Institute]

Astronomie
et espace

Aérodynamique et
moyens de transport

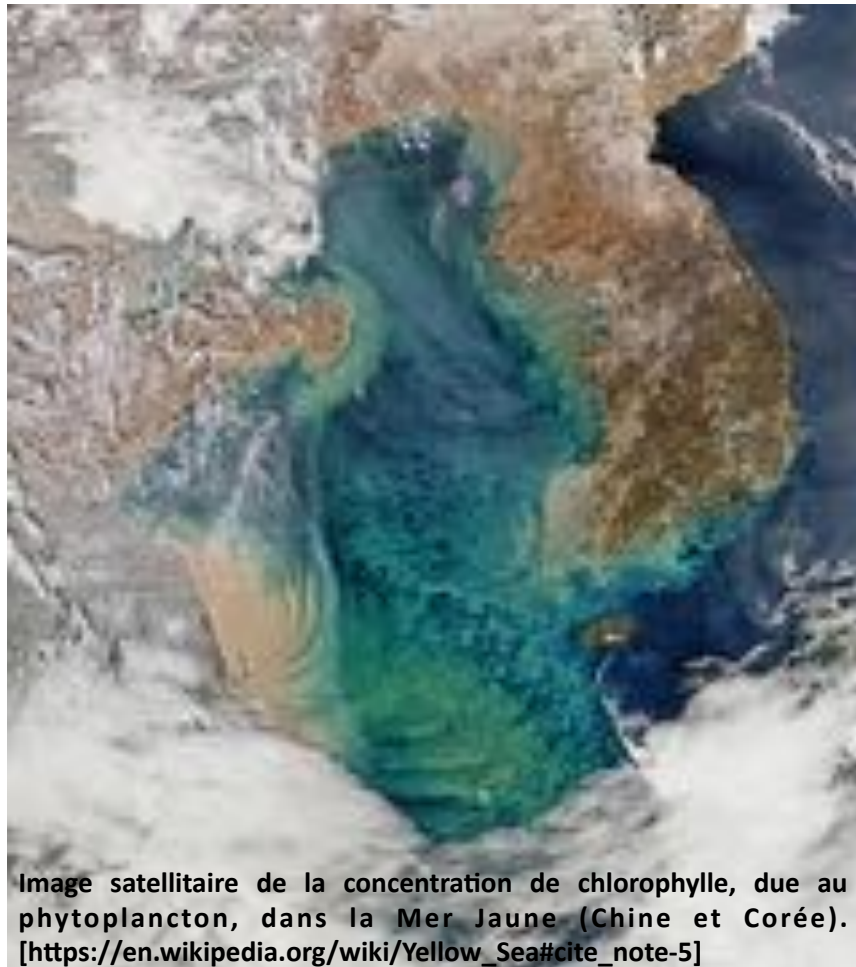


Image satellitaire de la concentration de chlorophylle, due au phytoplancton, dans la Mer Jaune (Chine et Corée).
[https://en.wikipedia.org/wiki/Yellow_Sea#cite_note-5]

Géophysique et
environnement



Production
d'énergie

Fluctuations turbulentes et perte d'énergie



Merci au Forum départemental des Sciences de Villeneuve d'Ascq pour la soufflerie de table et à P. Berti pour la voiture.

L'écoulement désordonné dans l'espace et dans le temps (turbulent) derrière la voiture provoque une augmentation de la traînée (frottement avec l'air) et réduit les performances aérodynamiques.

Ecoulements turbulents et propriétés de dispersion



Submesoscale dispersion in the vicinity of the *Deepwater Horizon* spill

Andrew C. Poje^a, Tamay M. Özgökmen^{b,1}, Bruce L. Lipphardt, Jr.^c, Brian K. Haus^b, Edward H. Ryan^b, Angelique C. Haza^b, Gregg A. Jacobs^d, A. J. H. M. Reniers^b, Maria Josefina Olascoaga^b, Guillaume Novelli^b, Annalisa Griffa^e, Francisco J. Beron-Vera^b, Shuyi S. Chen^b, Emanuel Coelho^d, Patrick J. Hogan^d, Albert D. Kirwan, Jr.^c, Helga S. Huntley^e, and Arthur J. Mariano^b

^aDepartment of Mathematics, Graduate Physics Program, College of Staten Island, City University of New York, New York, NY 10314; ^bDepartment of Ocean Sciences, Rosenstiel School of Marine and Atmospheric Sciences, University of Miami, Miami, FL 33176; ^cSchool of Marine Science and Policy, University of Delaware, Newark, DE 19718; ^dOcean Dynamics and Prediction, Naval Research Laboratory, Stennis Space Center, Hancock County, MS 39529; and ^eIstituto di Scienze Marine, Consiglio Nazionale delle Ricerche, 19032 La Spezia, Italy

Edited by Carl Wunsch, Harvard University, Cambridge, MA, and approved July 24, 2014 (received for review February 7, 2014)

Significance

We report here on results obtained from the largest upper-ocean dispersion field program conducted to date. The observations provided, for the first time to our knowledge, an accurate and nearly simultaneous description of the ocean surface velocity field on spatial scales ranging from 100 m to 100 km. We show conclusively that ocean flows contain significant energy at scales below 10 km and that their fluctuations dictate the initial spread of tracer/pollutant clouds. Neither state-of-the-art operational models nor satellite altimeters capture the flows needed for accurately estimating the dispersion of surface particles.

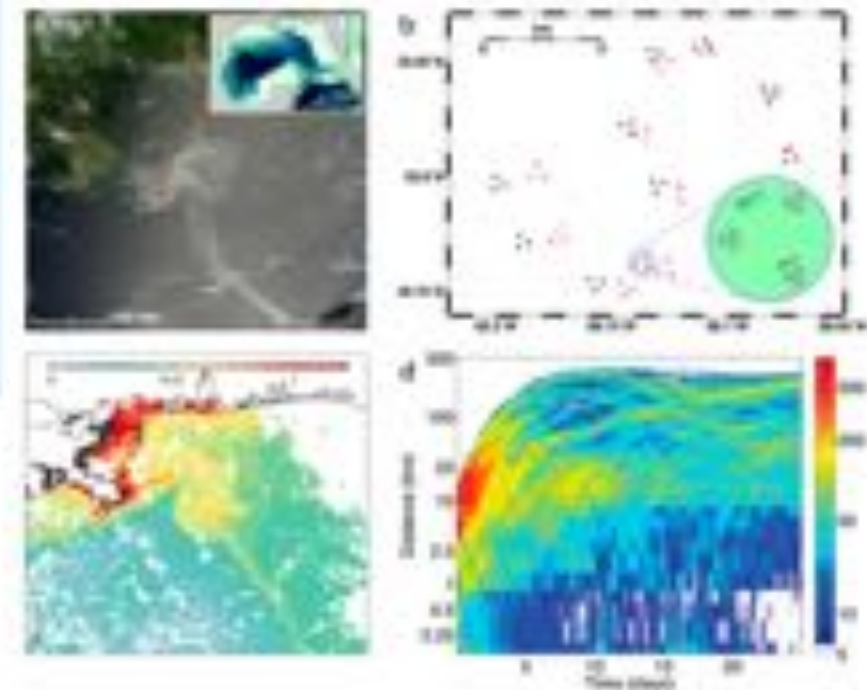
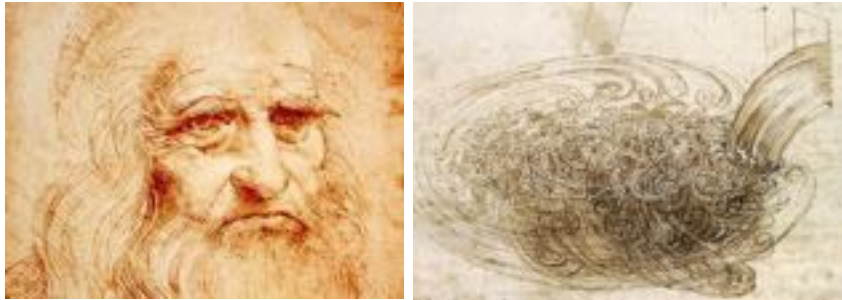


Fig. 1. Multiscale flows near the Deepwater Horizon region. (A) Synthetic aperture radar (SAR) image of the Deepwater Horizon oil rig taken on May 11, 2010. The red diamond marks the location of the (lost) wellhead, and the inset shows the geographic location. (B) Drifter launch patterns. The actual pattern obtained (red circles) for 21 at the launch time of the last drifter compared with the targeted (orange) (black circles). The inset shows a single node of the multiscale launch pattern. (C) Chlorophyll-a concentration (indicative of phytoplankton) suspended in the upper ocean (flow) derived from the moderate-resolution imaging spectrometry sensor aboard the Aqua satellite on July 12, 2013. The clarity of this image is a byproduct of the Aqua experiment sampled flow conditions similar to flows during the spill. (D) The time evolution of the number of drifter pairs at given separation distances for the 22 remote (open) number-of-log scale.

Deepwater Horizon oil spill
(Gulf of Mexico, 2010)



Quelques étapes historiques fondamentales



Leonardo da Vinci (1452-1519):

Premières descriptions des mouvements turbulents d'un fluide



Claude-Louis Navier (1785-1836) George Gabriel Stokes (1819-1903)

Equations fondamentales de la dynamique des fluides



Osborne Reynolds (1842-1912)

Expériences sur la transition à la turbulence et propriétés de mélange



Lewis Fry Richardson (1881-1953):

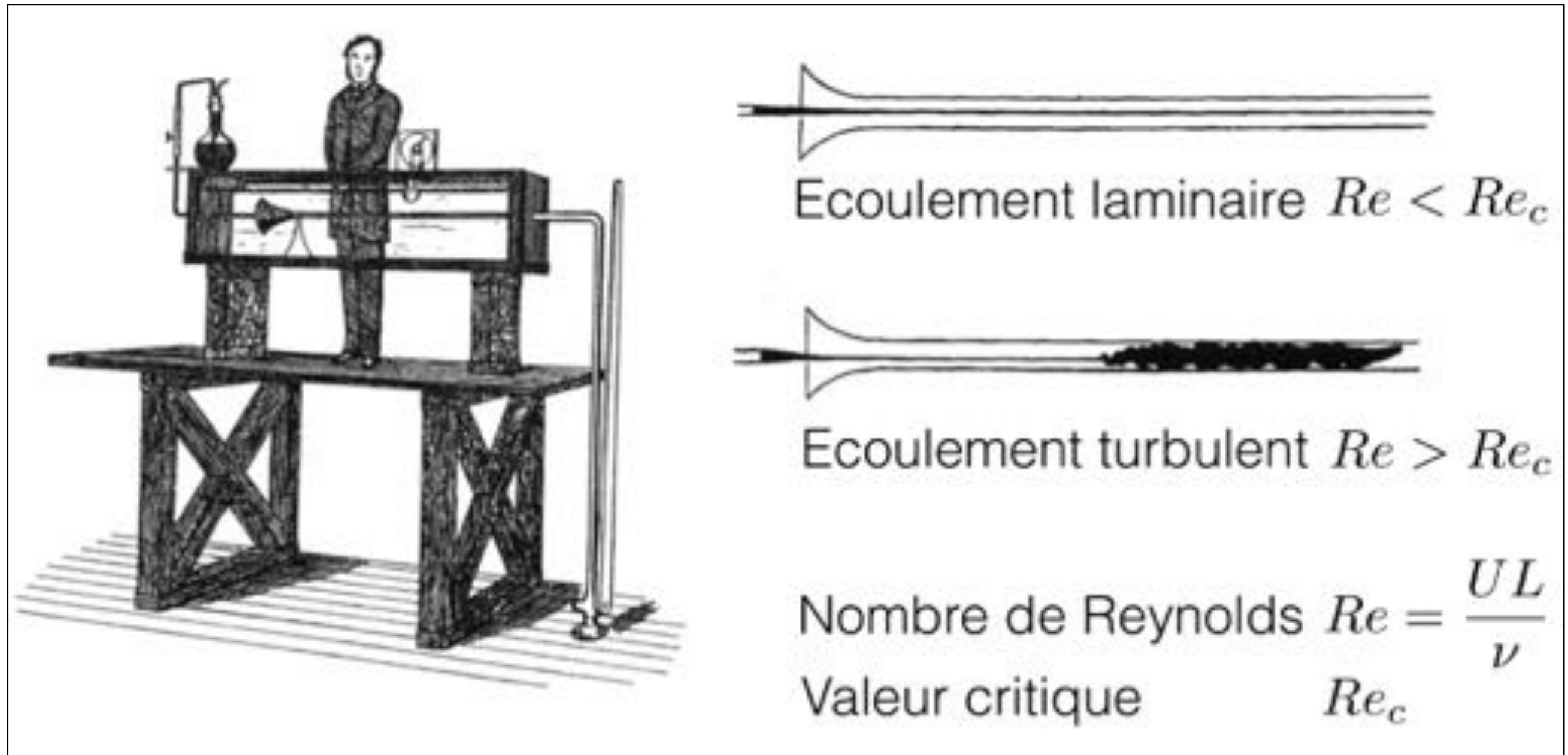
Approches statistiques et simulations numériques



Andrey Nikolaevich Kolmogorov (1903-1987)

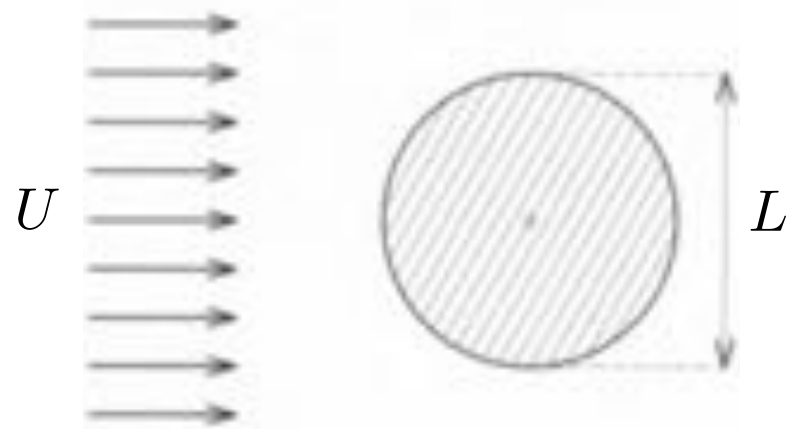
Théorie statistique de la turbulence et universalité (K41, K62)

Transport et mélange dans un écoulement turbulent



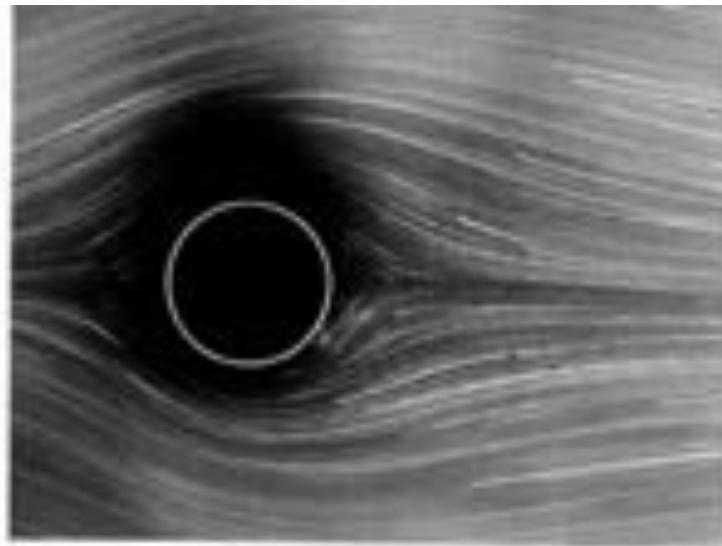
Expérience de Reynolds (1883): transition à la turbulence dans une conduite.
[Adaptation de https://en.wikipedia.org/wiki/Osborne_Reynolds]

Transition à la turbulence

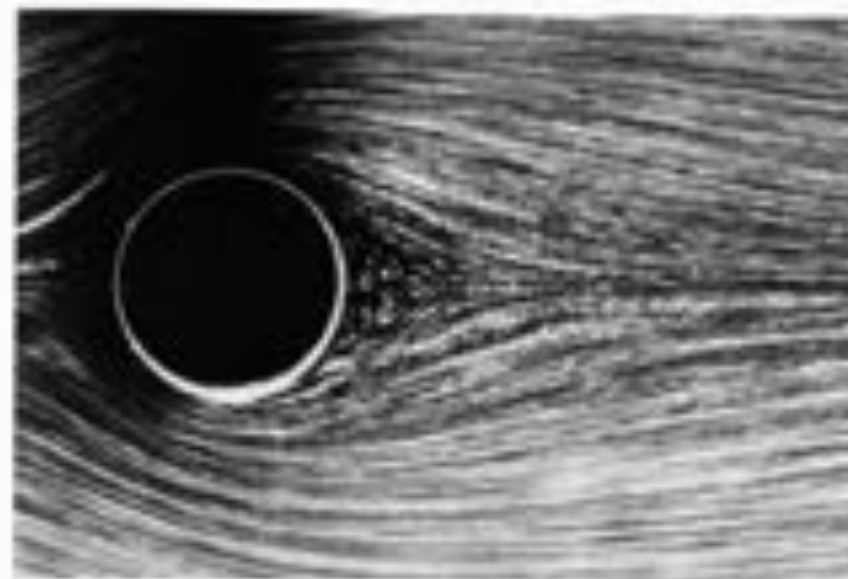


$$Re = \frac{UL}{\nu}$$

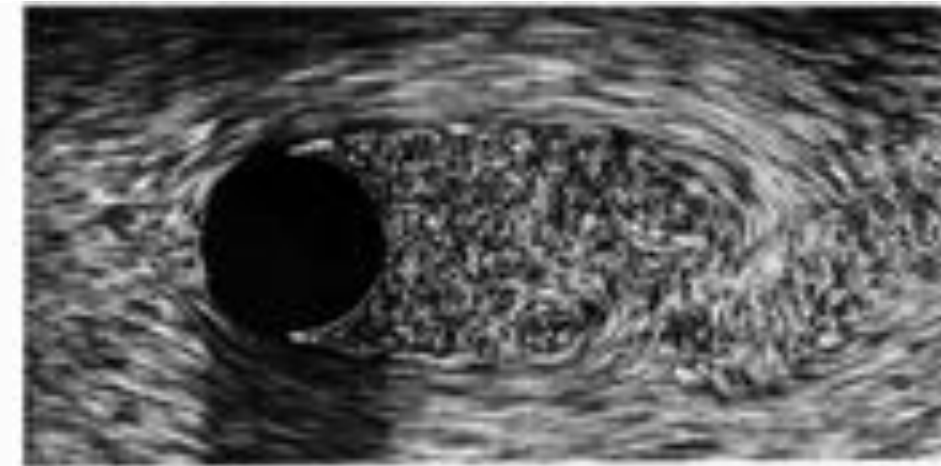
Nombre de Reynolds: rapport entre forces d'inertie et forces de viscosité



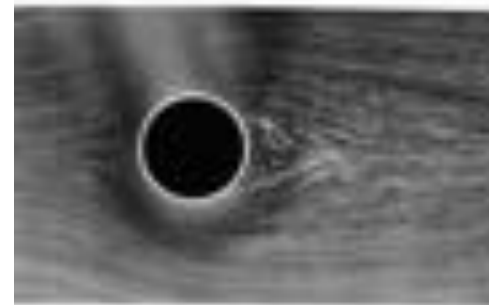
35. Circular cylinder at $Re = 5$. At this Reynolds number the structure around the cylinder has the form of a single vortex. The flow is symmetric about the axis. The image is about 5-7.



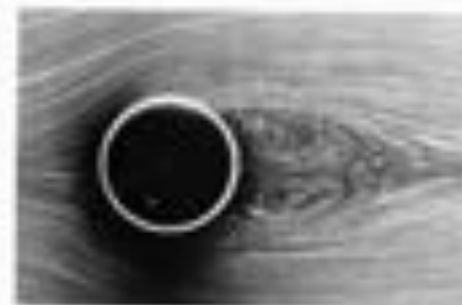
40. Circular cylinder at $Re = 10$. The flow is still symmetric about the axis. The wake is slightly more complex than at $Re = 5$. The image is about 10-12.



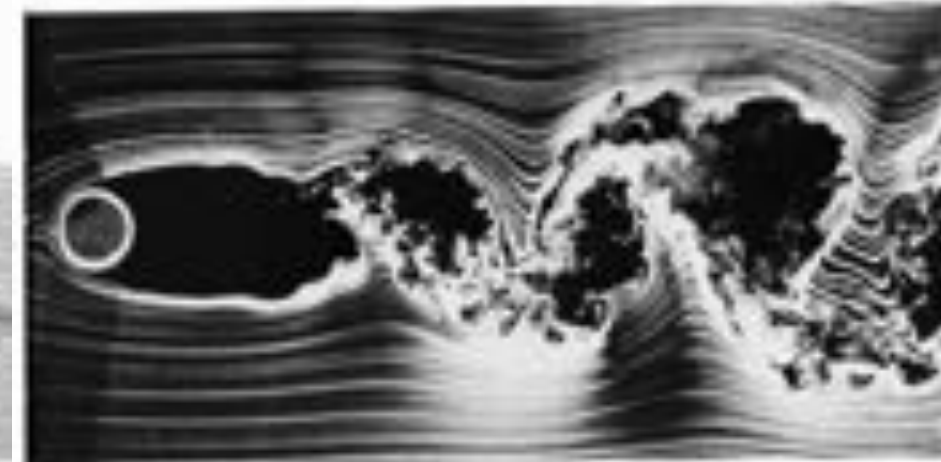
45. Circular cylinder at $Re = 20$. The flow is asymmetric about the axis. The wake is highly complex. The image is about 20-25.



50. Circular cylinder at $Re = 30$. The flow is highly asymmetric about the axis. The wake is very complex. The image is about 30-35.



55. Circular cylinder at $Re = 40$. The flow is highly asymmetric about the axis. The wake is very complex. The image is about 40-45.



60. Circular cylinder at $Re = 50$. The flow is highly asymmetric about the axis. The wake is very complex. The image is about 50-55.

Van Dyke, An album of fluid motion,
The Parabolic Press (1982)

Dynamique des fluides incompressibles

masse x accélération = force

Principe fondamental de la dynamique (Newton)

Par unité de volume: équation de Navier-Stokes:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P + \nu \Delta \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

condition d'incompressibilité

$$\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$$

Vitesse du fluide au point (x,y,z) au temp t

Equation de Navier-Stokes

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P + \nu \Delta \mathbf{u} + \mathbf{f}$$

↑
Forces de viscosité


Terme **nonlinéaire**/inertiel/convectif

Il est *dominant* par rapport au terme de viscosité *quand le nombre de Reynolds est suffisamment grand*. Il décrit les interactions de la vitesse avec elle même et il est le **responsable de la complexité** des écoulements turbulents.


The Clay Mathematical Institute Millennium problems

1.000.000 \$ prize

Do solutions of the Navier-Stokes equations exist
for any well-behaved initial condition?
Are they unique?

[ABOUT](#) [PROGRAMS](#) [MILLENNIUM PROBLEMS](#) [PEOPLE](#) [PUBLICATIONS](#) [EVENTS](#) [EUCLID](#)

Navier–Stokes Equation



Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

Image: Sir George Gabriel Stokes (13 August 1819–1 February 1903). [Public Domain](#)

This problem is: Unsolved

Rules:

[Rules for the Millennium Prizes](#)

Related Documents:

[Official Problem Description](#)

Related Links:

[Lecture by Luis Caffarelli](#)

Tourbillons

Au centre des approches théoriques récentes (20ème siècle) visant à la compréhension de la turbulence il y a le concept d'interaction (via le terme convectif nonlinéaire) entre tourbillons de différente taille et des échanges d'énergie associés.



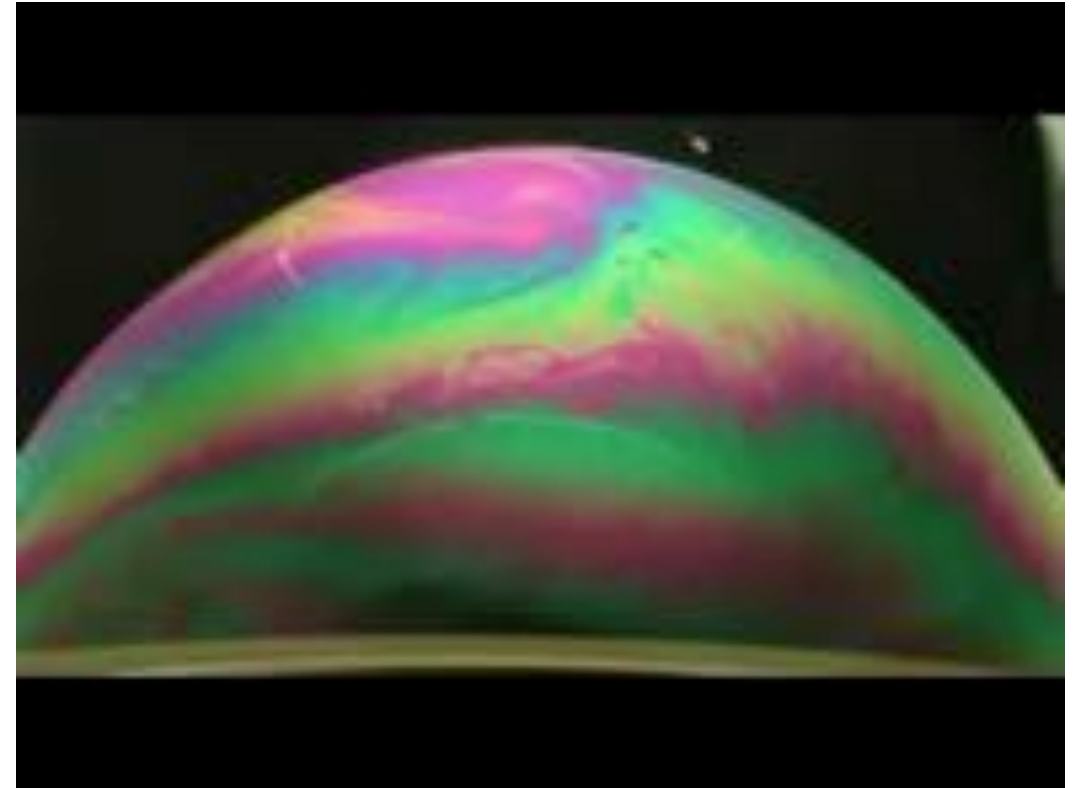
Un tourbillon dans une bouteille (expérience maison)

Tourbillons en interaction

Dans des expériences dans des films de savon:

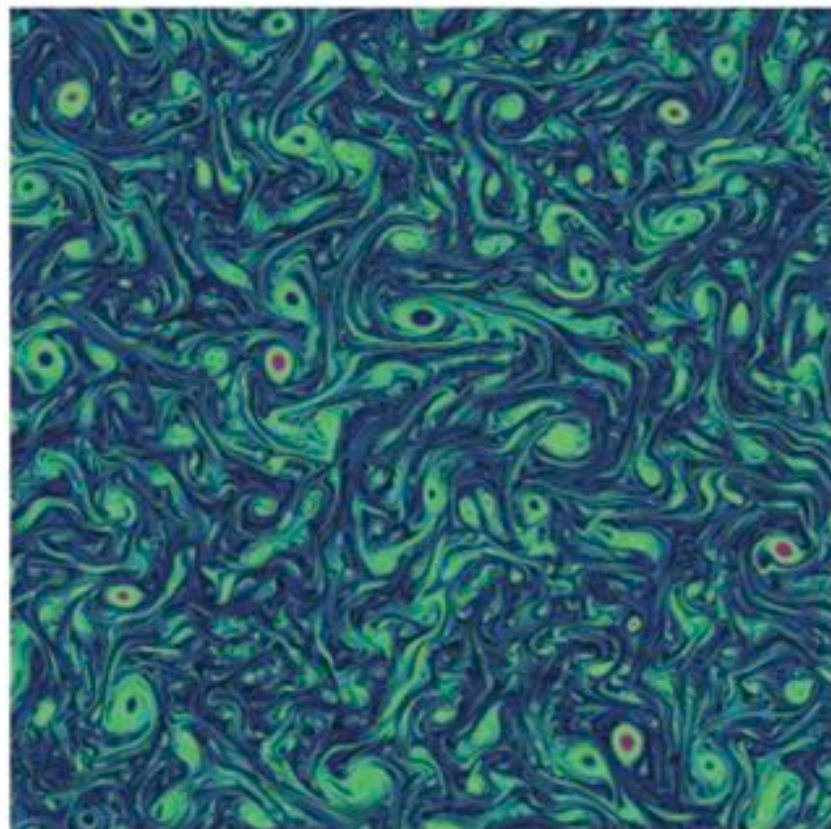


Expérience maison



Soap bubble submitted to temperature variations similar to those found between the equator and the pole of the Earth.

F. Seychelles, Y. Amarouchene, M. Besafi, H. Kellay, Phys. Rev. Lett. **100**, 144501 (2008)



Positive
0
Negative
Vorticity

← Dans des simulations numériques

Visualisation d'un écoulement turbulent bidimensionnel.

G. Boffetta, R. Ecke, Annu. Rev. Fluid Mech. **44**, 427 (2012)

Approches statistiques: cascade turbulente



Lewis Fry Richardson (1881-1953)

To following historical and poetical note serves as concluding note about cascades. The modern concept of a cascade probably owes its origins to Lewis Fry Richardson (1922). He took inspiration from observations of clouds and from Jonathan Swift's verse:

So, nat'ralists observe, a flea
Hath smaller fleas that on him prey;
And these have smaller yet to bite 'em,
And so proceed *ad infinitum*.
Thus every poet, in his kind,
Is bit by him that comes behind.

The last two lines, which are not usually quoted, may also be relevant, if 'fluid dynamicist' is substituted for 'poet'.

104 Phenomenology of turbulence in the sense of Kolmogorov 1941

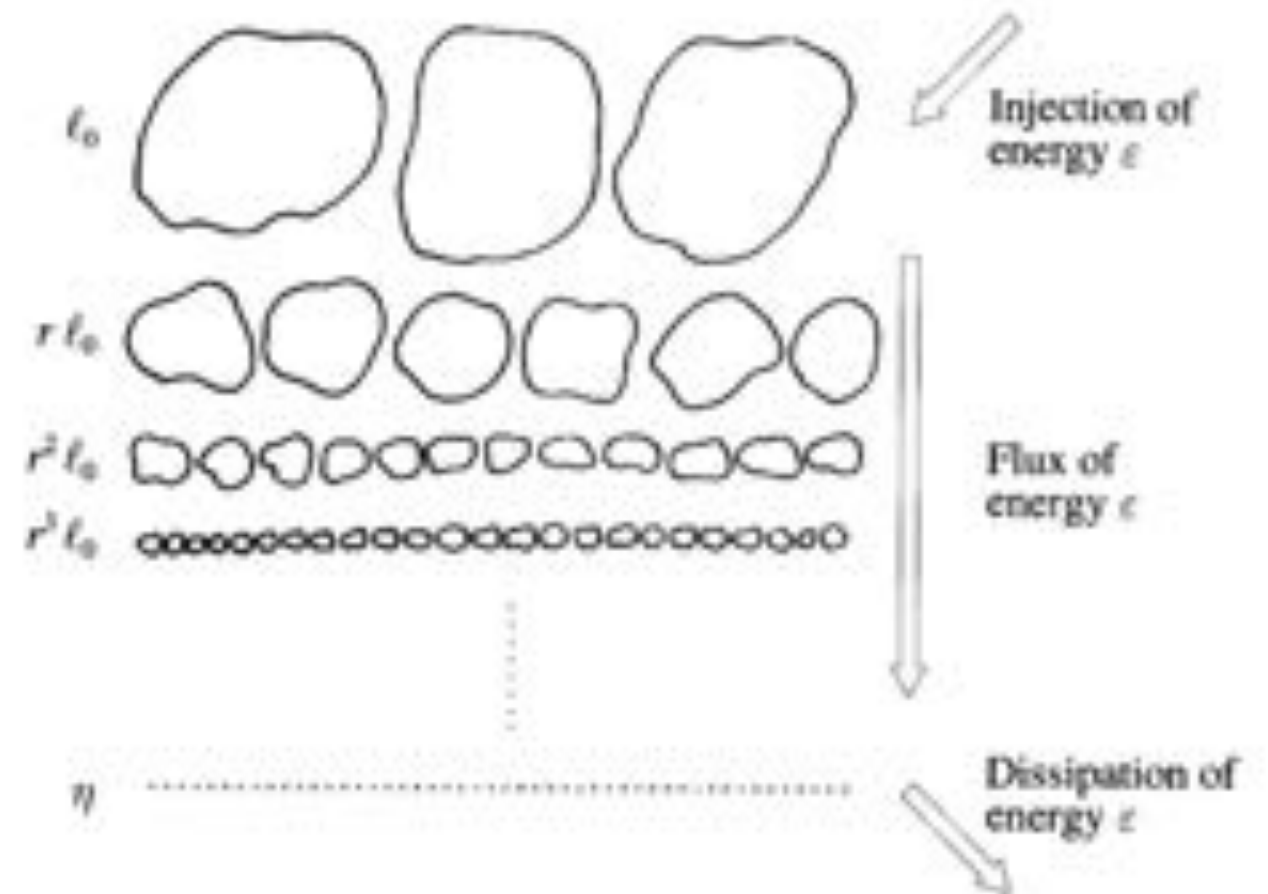
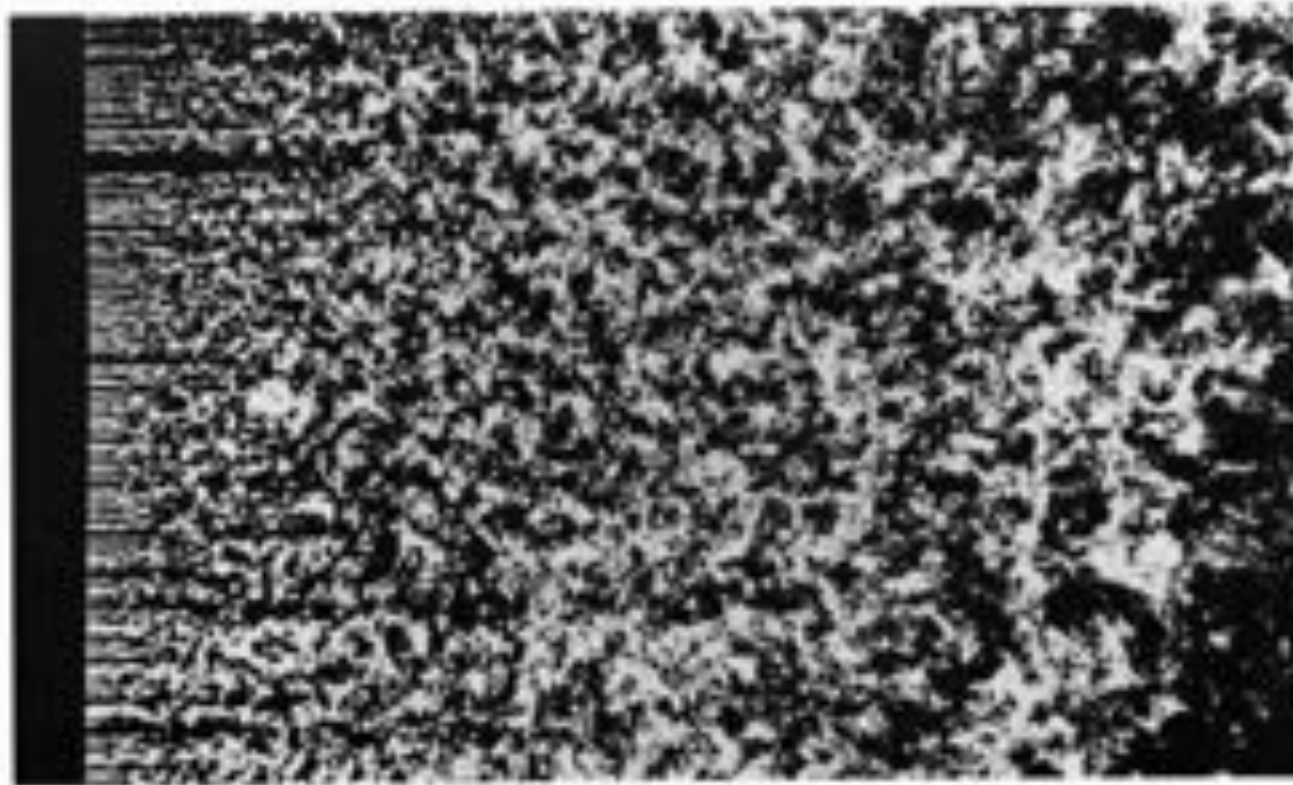


Fig. 7.2. The cascade according to the Kolmogorov 1941 theory. Notice that at each step the eddies are space-filling.

Turbulence homogène isotrope

Expérience

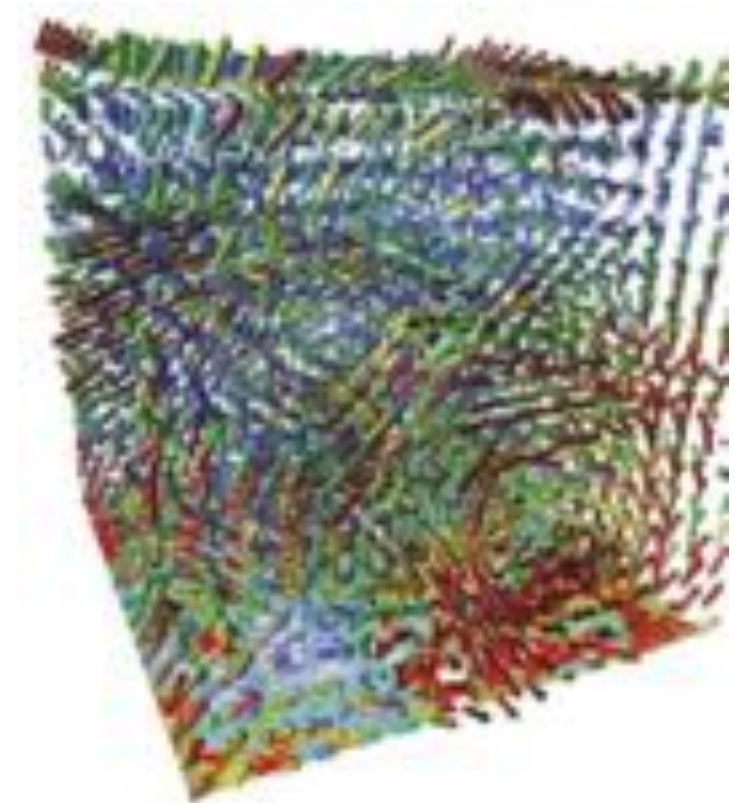


153. Homogeneous turbulence behind a grid. Behind a finer grid than above, the mixing unstable wakes quickly form a homogeneous field. As it decays down-

stream, it provides a useful approximation to the idealization of isotropic turbulence. Photograph by Thomas Carlier and Hassan Nagib

Van Dyke, An album of fluid motion,
The Parabolic Press (1982)

Simulation numérique



LA CONFIGURAZIONE ISTANTANEA DEL CAMPO DI VELOCITÀ di un fluido turbolento calcolato con una simulazione numerica su una griglia cubica di 1024 punti di lato. Le frecce indicano l'intensità e la direzione del campo in un sottoinsieme di punti. Il numero di Reynolds è stimato a partire dal volume V occupato dal fluido, dall'intensità tipica del campo di velocità U (dato dalla media dell'ampiezza delle frecce su tutto il volume) e dalla viscosità molecolare del fluido in questione (per l'acqua a temperatura ambiente $\nu = 1,4 \times 10^{-6} \text{ m}^2/\text{s}$). Negli esperimenti di laboratorio, la turbolenza sviluppata ad alti numeri di Reynolds si ottiene aumentando le dimensioni dell'apparato oppure l'intensità delle forze esterne che agiscono il moto. Nelle simulazioni invece è più semplice tenere invariati la geometria e il tipo di forze esterne ed aumentare la viscosità molecolare.

Biferale, Boffetta, Celani, Lanotte, Toschi,
Le Scienze **443**, 2-9 (2005) [Science &
Vie, version italienne]

Une théorie de la turbulence



Andrey Nikolaevich
Kolmogorov (1903-1987)

Théorie phénoménologique de Kolmogorov (1941)

Universalité à petite échelle

Formalisation mathématique de la cascade turbulente

Approche probabiliste

Prédictions en sense statistique

Prédictions spectre énergie cinétique (répartition d'énergie entre les tourbillons de différente taille)

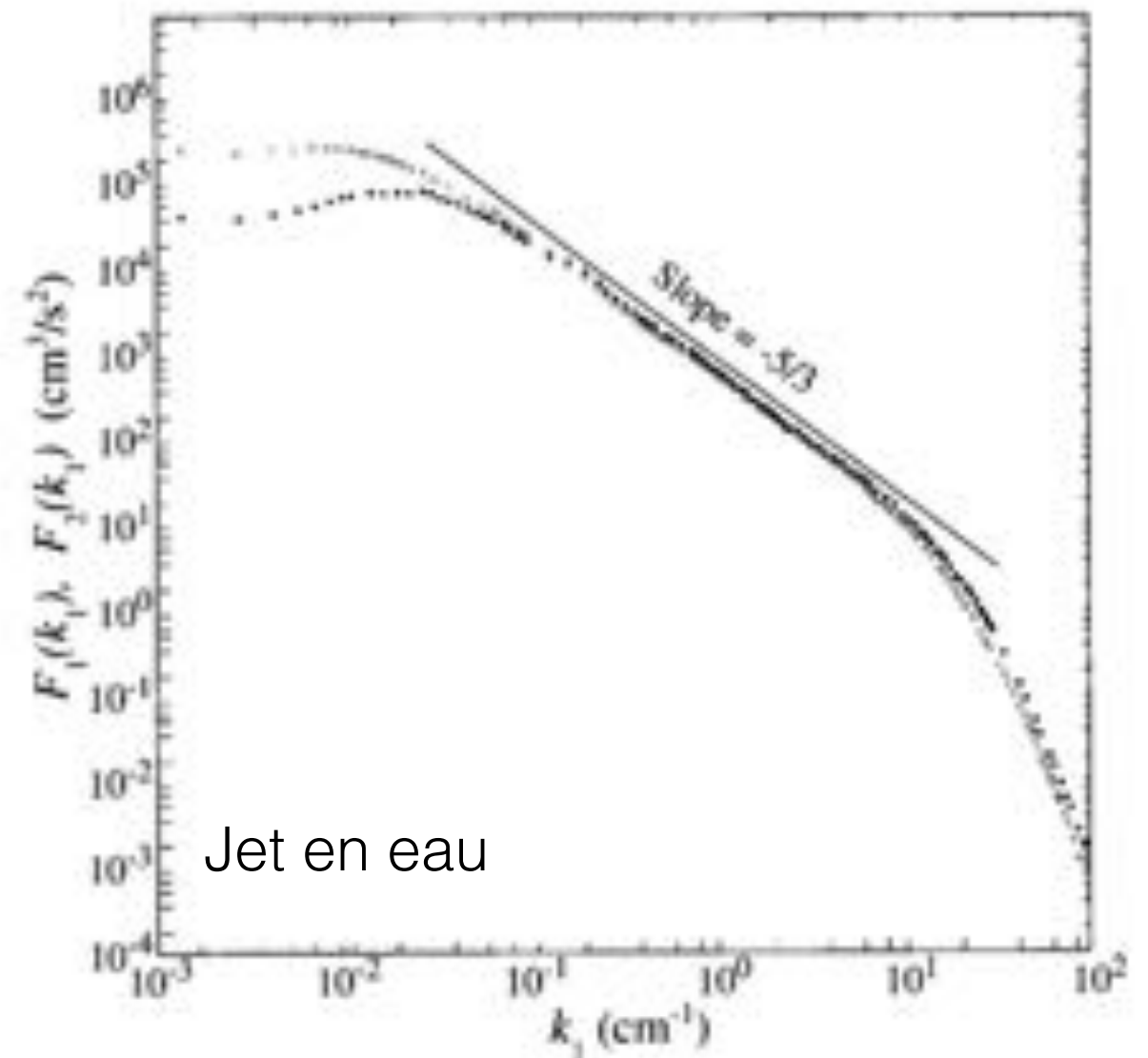
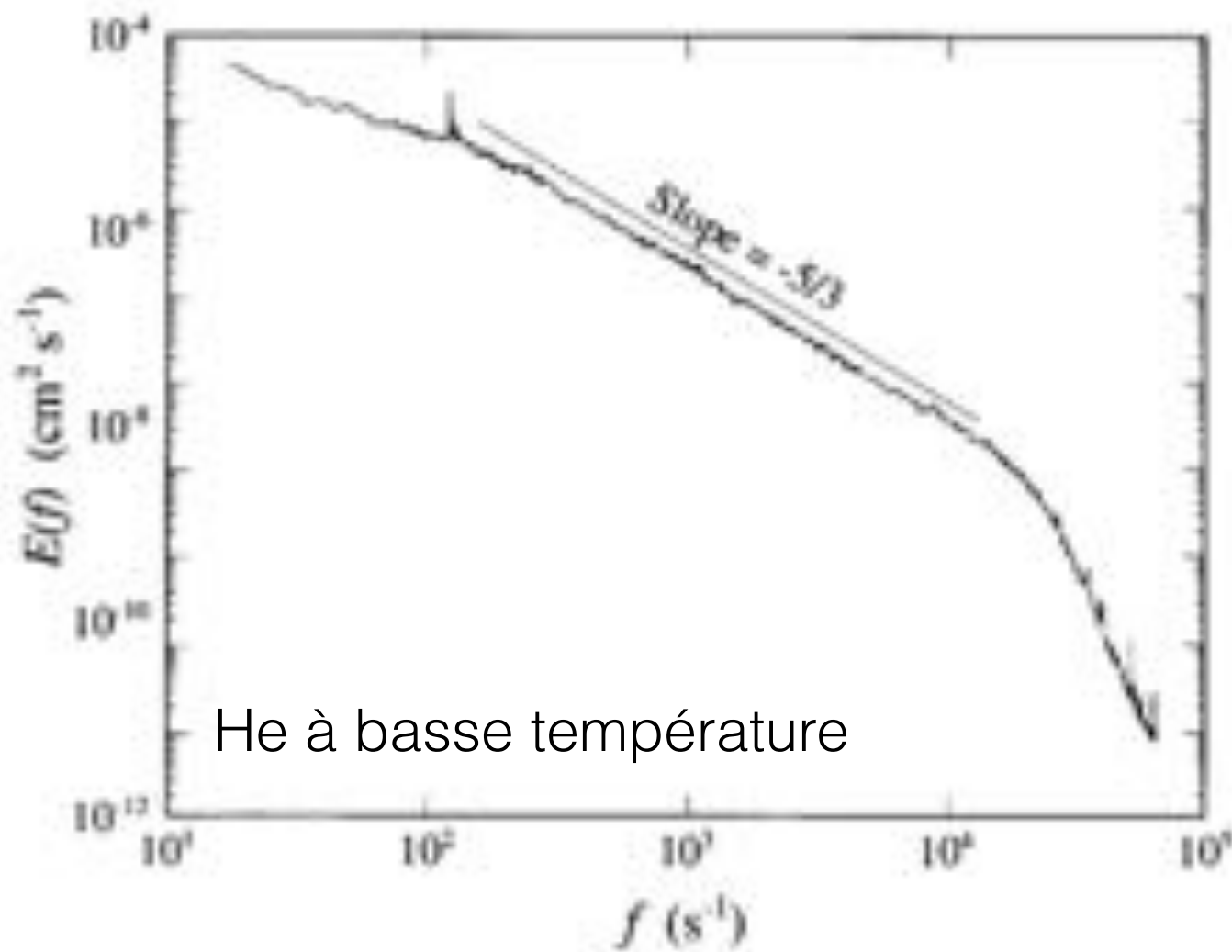
Idées novatrices: invariance d'échelle

Applications dans d'autres domaines
(géométrie fractale, transition de phase)

Spectre d'énergie cinétique

Prédiction théorique (universelle): $E(k) = C\epsilon^{2/3}k^{-5/3}$

Vérifications expérimentales:



... et plusieurs autres systèmes

Simulations numériques

A partir des années récentes les simulations numériques se sont révélées aussi un outil incontournable pour explorer la turbulence.



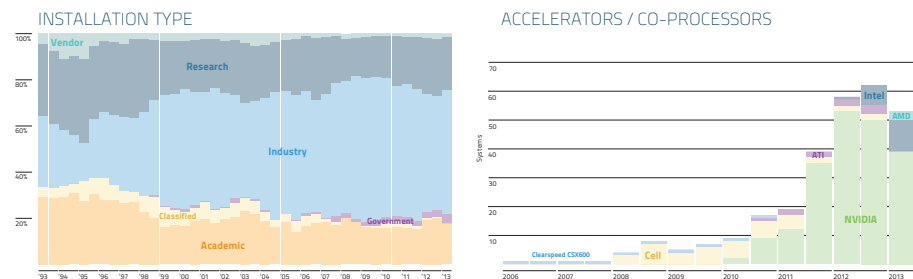
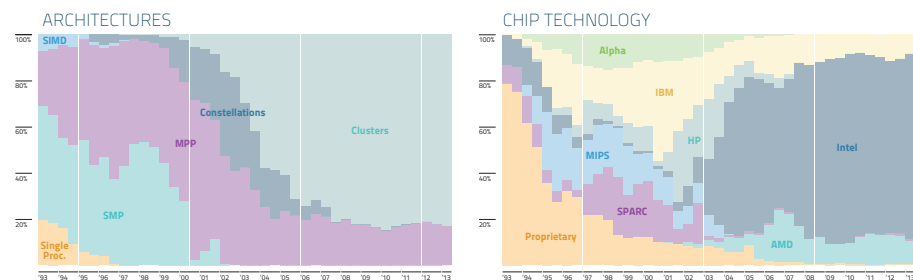
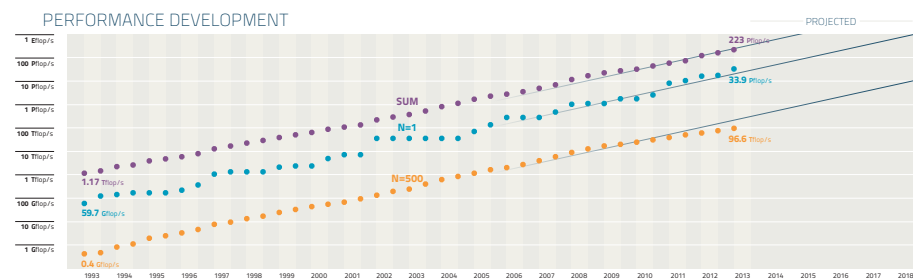
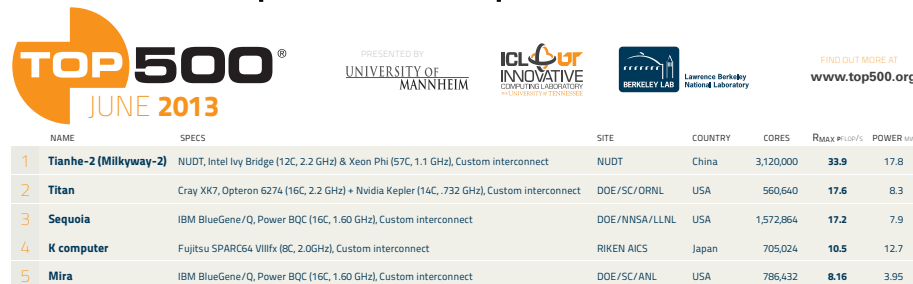
Weather prediction by numerical methods

Richardson foresaw a “forecast factory,” where he calculated that 64,000 human “computers,” each responsible for a small part of the globe, would be needed to keep “pace with the weather” in order to predict weather conditions. They would be housed in a circular hall like a theater, with galleries going around the room and a map painted on the walls and ceiling. A conductor located in the center of the hall would coordinate the calculations using colored lights...

While Richardson’s vision never became a reality, the use of mathematics to predict the weather did develop over the years.

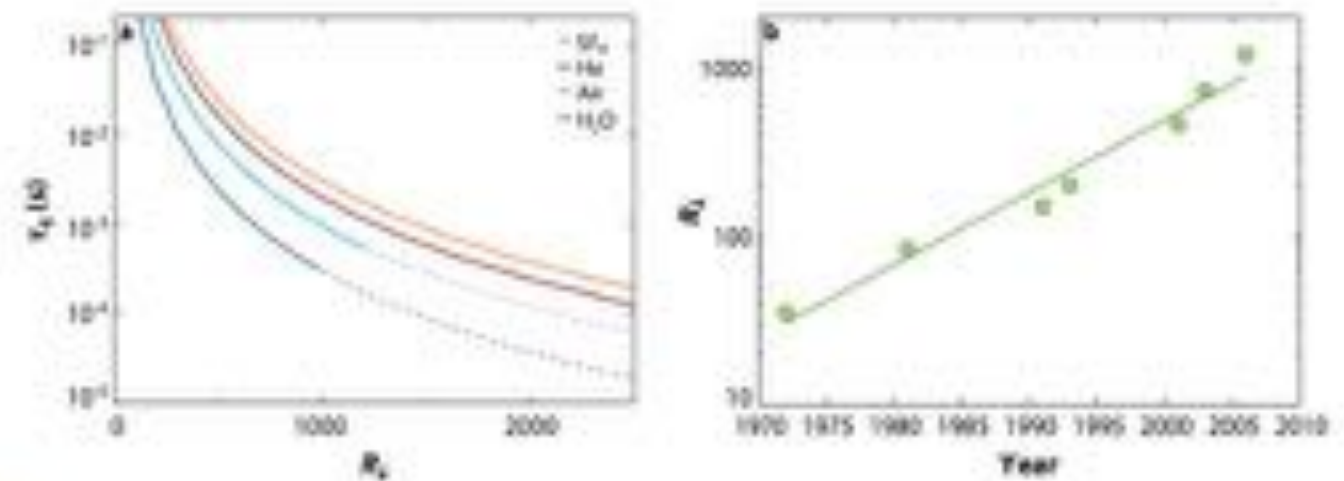
Simulations numériques

Supercomputers



HPLINPACK A Portable Implementation of the High Performance Linpack Benchmark for Distributed Memory Computers MORE INFO AT <http://icl.utk.edu/hpl/>

Turbulence numérique



Toschi, Bodenschatz, Annu. Rev. Fluid Mech., **41**, 375–404 (2009)

Mesures précises de quantités inaccessibles dans les expériences de laboratoire
Contrôle des prédictions théoriques, indications pour les développements futurs

Entry #: 84174

Vortices within vortices: hierarchical nature of vortex tubes in turbulence

Kai Bürger¹, Marc Treib¹, Rüdiger Westermann¹,
Suzanne Werner², Cristian C Lalescu³,
Alexander Szalay², Charles Meneveau⁴, Gregory L Eyink^{2,3,4}

¹ Informatik 15 (Computer Graphik & Visualisierung), Technische Universität München

² Department of Physics & Astronomy, The Johns Hopkins University

³ Department of Applied Mathematics & Statistics, The Johns Hopkins University

⁴ Department of Mechanical Engineering, The Johns Hopkins University

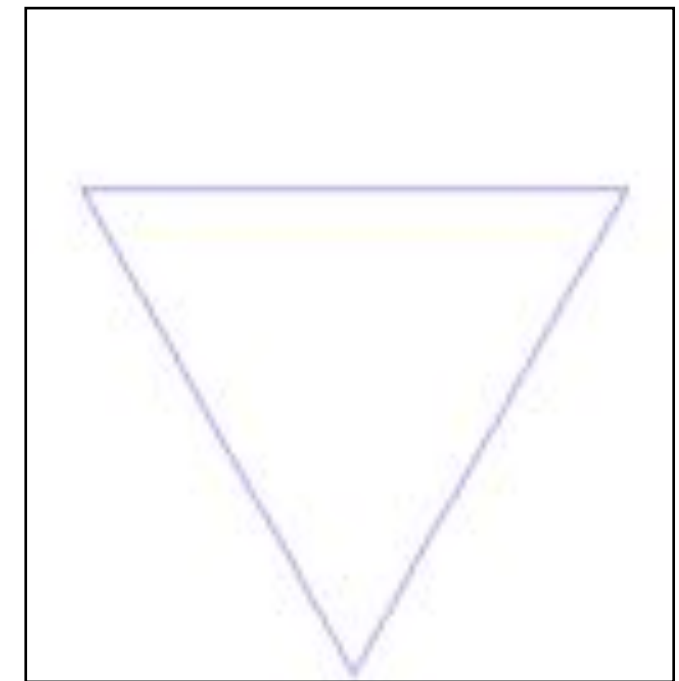
Invariance d'échelle de l'écoulement

Invariance d'échelle et géométrie fractale

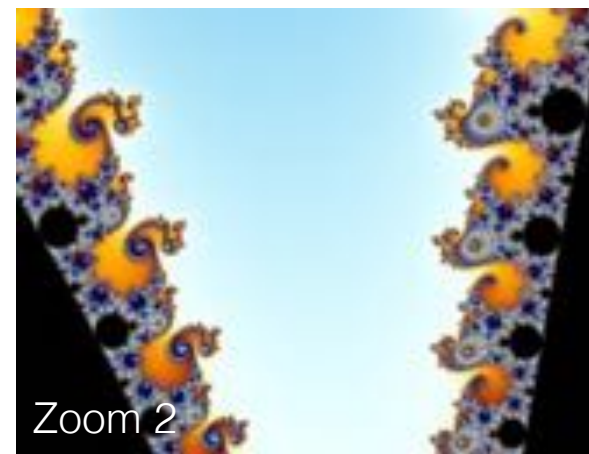
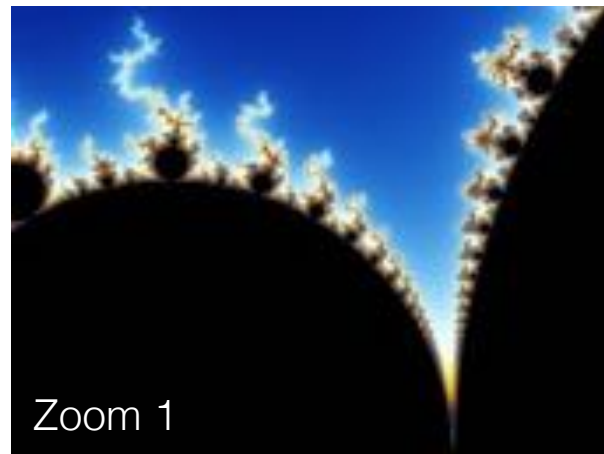
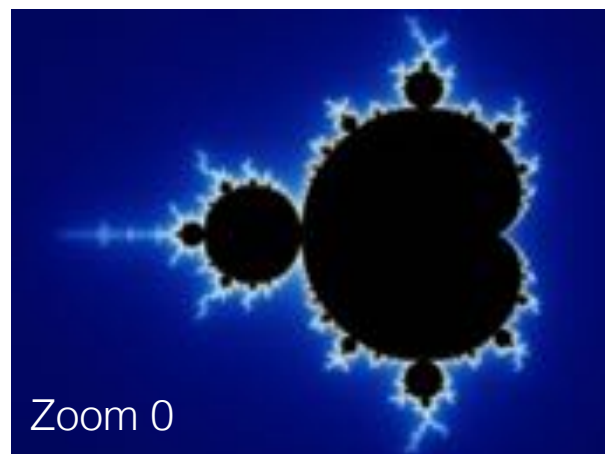


Une figure **fractale** est un objet mathématique, telle une courbe ou une surface, dont la structure est invariante par changement d'échelle.

Les fractales peuvent être définies à l'image des poupées russes qui renferment une figurine identique à l'échelle près.



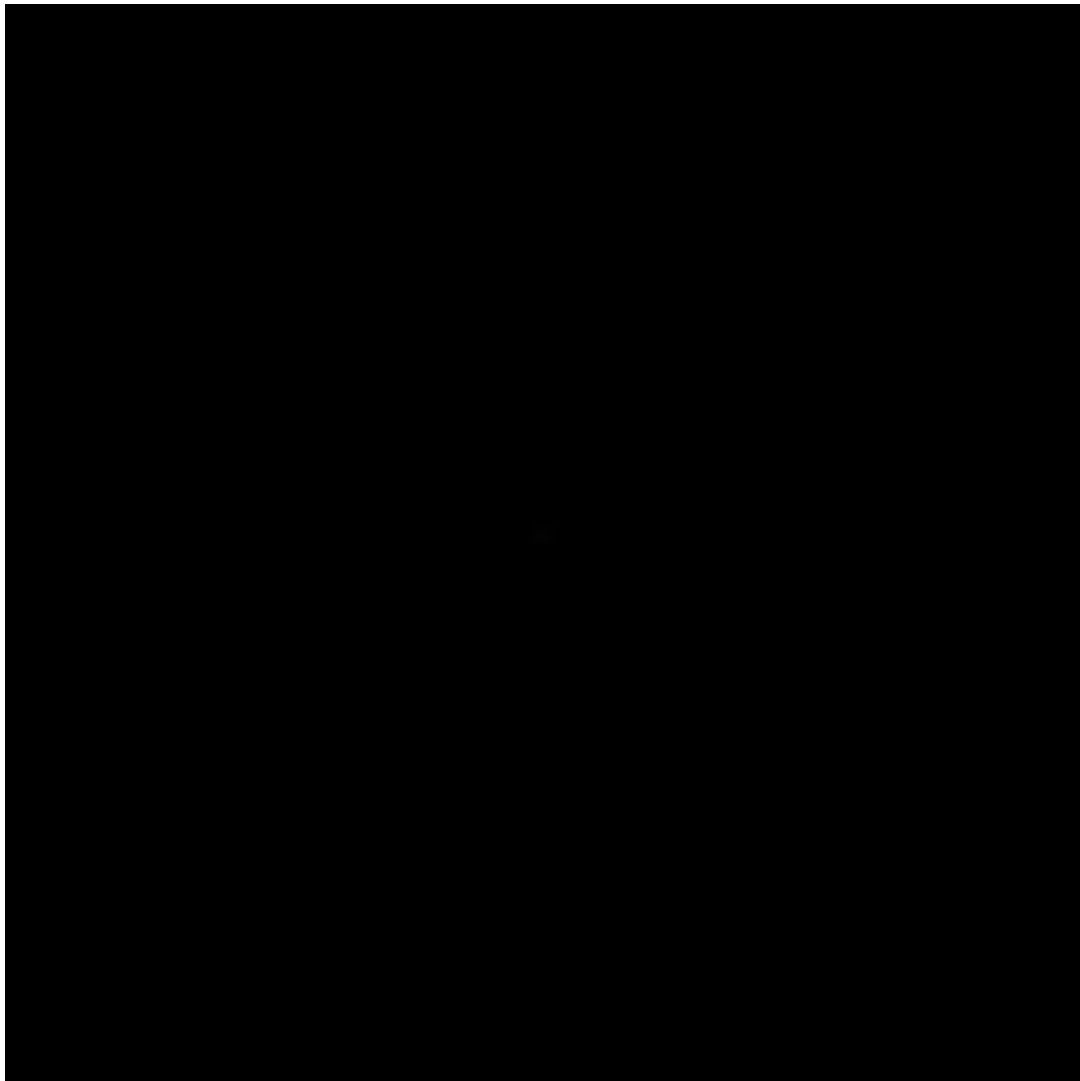
Courbe de von Koch
(flocon de neige)



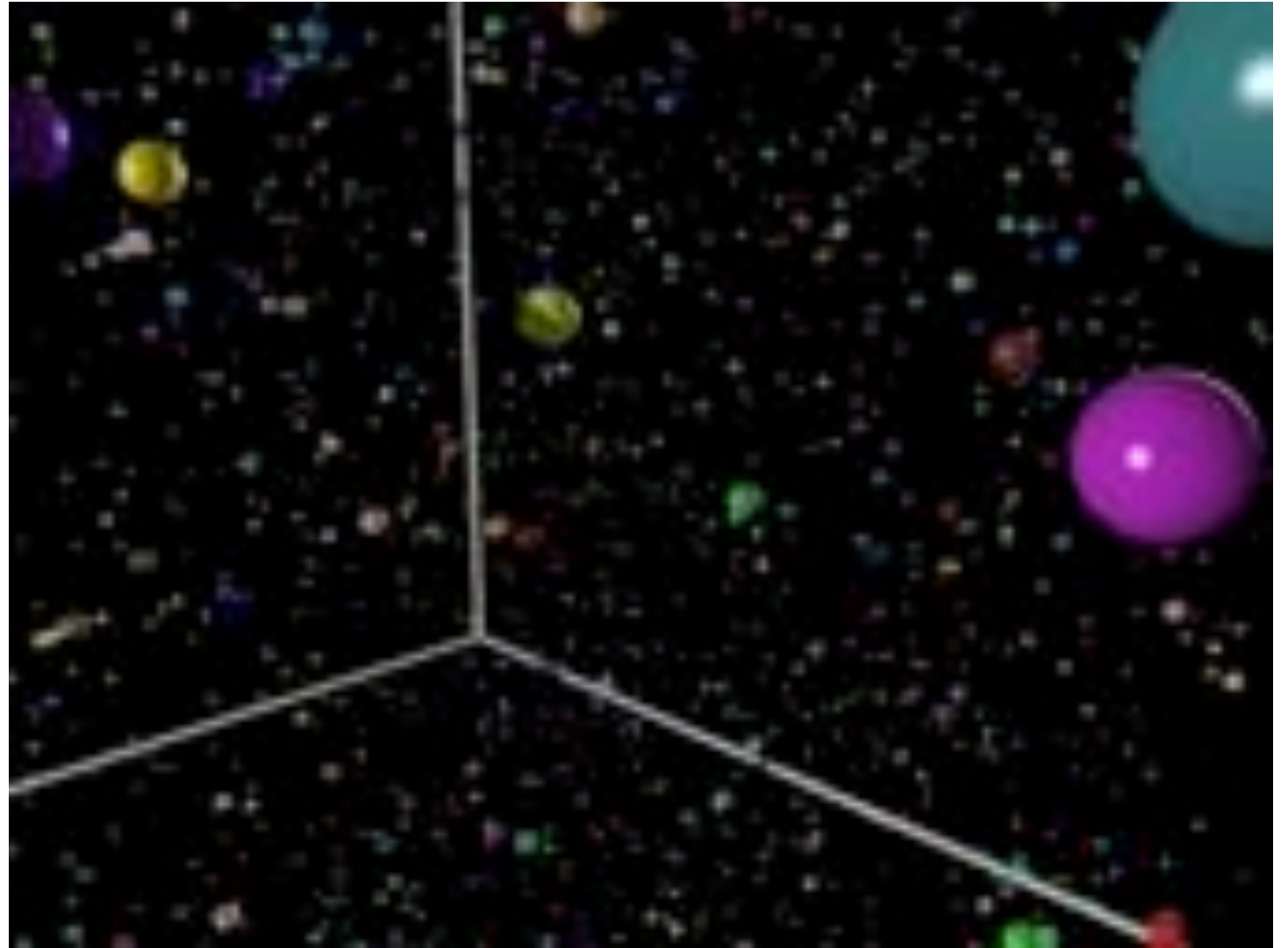
Ensemble de Mandelbrot

Dispersion turbulente et approches lagrangiennes

Augmentation efficacité du mélange (par agitation du fluide)



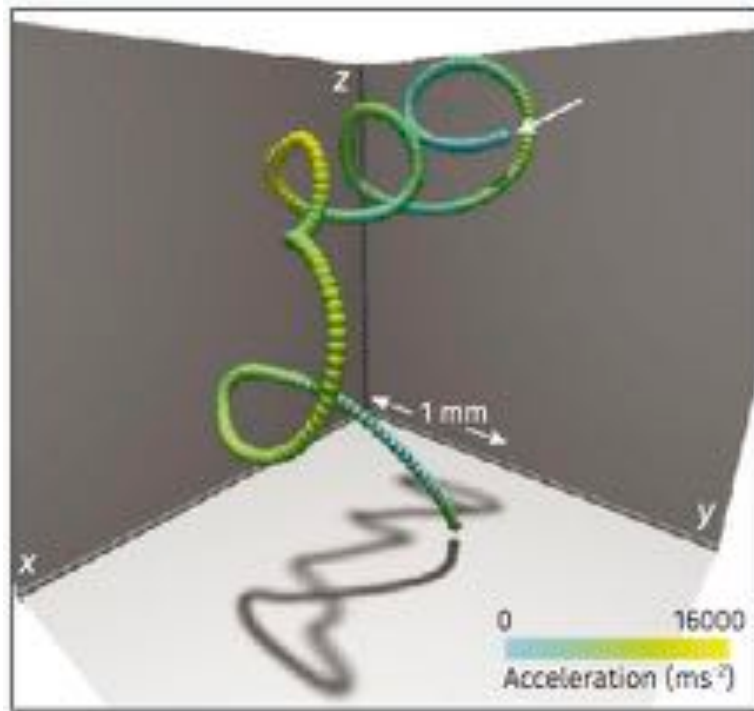
Point de vue lagrangien (d'une particule transportée par le fluide)



Diffusion turbulente: $R^2 \sim t^3$

Sans écoulement: $R^2 \sim t$

Dispersion turbulente



Expériences lagrangiennes: trajectoire à haute résolution et statistique de l'accélération turbulente. [A. La Porta et al. Nature **409**, 1017 (2001); G. A. Voth et al. J. Fluid mech **469**, 121 (2002)]

Les approches lagrangiennes ont permis de mettre en évidence l'importance des **événements extrêmes**.

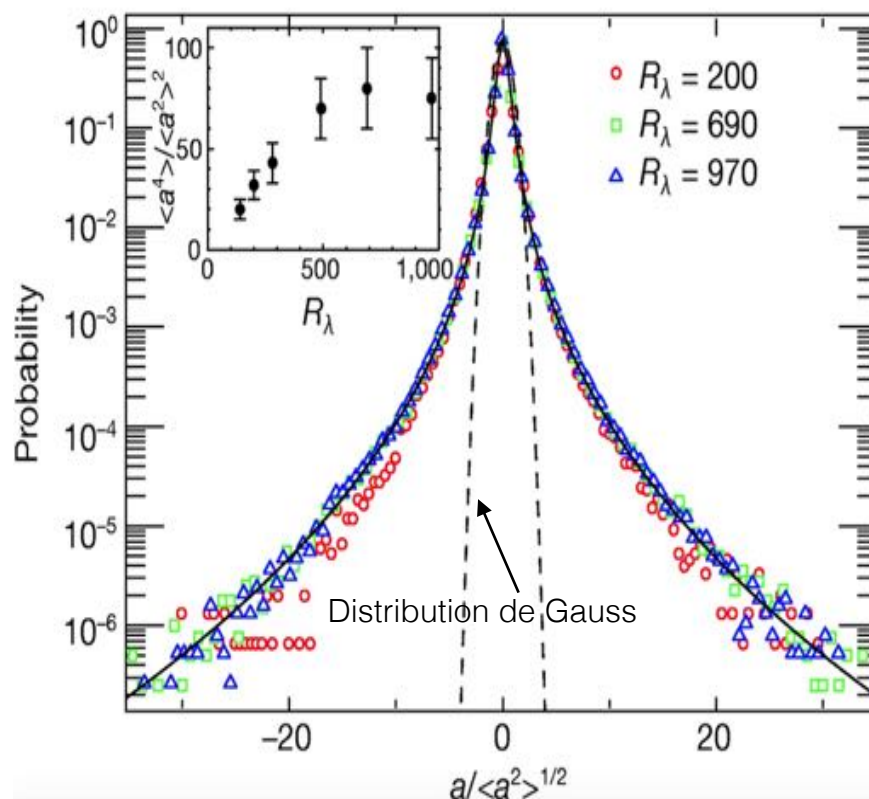
Valeurs extrêmes des fluctuations d'accélération (aux plus petites échelles) plus probables que prévu par une distribution normale (de Gauss): $a \approx 80 \sigma$.

A l'échelle d'un insecte: $a \approx 100 g$ pour une brise de $v = 20 \text{ km/h}$.

Origine: structures persistentes (filaments de vorticit , des cyclones miniaturis s).

Forces responsables: d pression par rapport au fluide environnant

Derri re un aspect d sordonn  et un caract re al atoire apparent il se cache un ordre (statistique) insoup onn .



Applications de ces connaissances: d veloppement de mod les math matiques simplifi s   utiliser dans les applications environnementales (p.ex. pour calculer la dispersion de polluants dans l'atmosph re).

Biferale, Boffetta, Celani, Lanotte, Toschi, Le Scienze **443**, 2-9 (2005) [Science & Vie, version italienne]

Conclusions

Loin d'être une exception, la **turbulence** des fluides est un phénomène très largement répandu, avec un **impact** sur nombreux problèmes techniques et scientifiques: *production de l'énergie, performance des moyens de transport, environnement, climat, formation des planètes, ...*

De par sa nature multi-échelle (présence de structures spatiales de tailles très différentes) et irrégulière en temps la turbulence représente un paradigme de **système complexe**.

Sa compréhension fine est encore *un des gros problèmes ouverts en physique*.

Les avancées dans le domaine sont étroitement reliées à celles en *mathématiques*, (équations différentielles, probabilité et statistique, calcul numérique), ainsi qu'aux développements technologiques (informatique, capteur à haute résolution).