Phenomenology of elastic turbulence in 2D polymer solutions

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Softflow 2009 - Complex and biofluids
Cargese, France

Outline

1. Elastic turbulence
2. Hydrodynamic model
3. DNS of elastic turbulence
4. Conclusions

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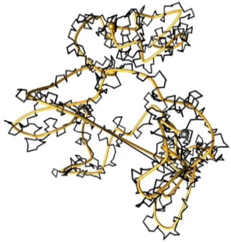
Viscoelastic fluids

Solutions of flexible long-chain polymers



$C \sim (1 \div 10) ppm$ in weight (dilute solutions)

$\tau \sim (1 \div 10) s$ (slowest) relaxation time

Striking effects on flowing fluids

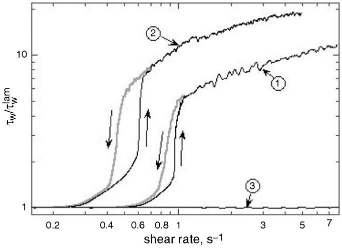
$Wi \equiv \frac{U\tau}{L}$ Weissenberg number

$Re \equiv \frac{UL}{\nu}$ Reynolds number

$Re \simeq 0; \; Wi \gg 1 \; \Rightarrow$ Elastic turbulence

Some experimental results

Flow resistance



- swirling flow between two parallel disks
- high m.w. polyacrylamide
- dilute solution in a viscous sugar syrup

A. Groisman, V. Steinberg, Nature **405**, 53 (2000)

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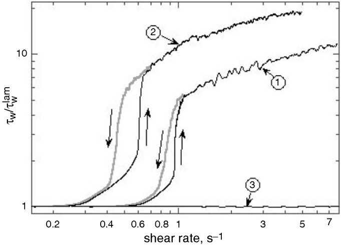
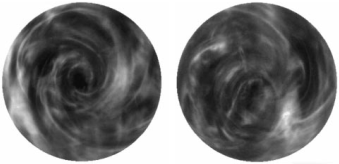


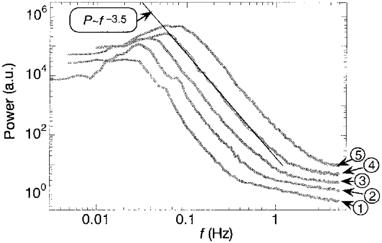
- swirling flow between two parallel disks
- high m.w. polyacrylamide
- dilute solution in a viscous sugar syrup

Snapshots of the flow

$Wi = 13$
$Re = 0.7$



Spectra of velocity fluctuations



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Oldroyd-B model

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu\Delta\boldsymbol{u} + \boldsymbol{f} + \underbrace{\frac{2\eta\nu}{\tau}\boldsymbol{\nabla}\cdot\boldsymbol{\sigma}}_{\textit{feedback}} \\ \partial_t \boldsymbol{\sigma} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{\sigma} = \underbrace{(\boldsymbol{\nabla}\boldsymbol{u})^T\cdot\boldsymbol{\sigma} + \boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}\boldsymbol{u})}_{\textit{stretching}} - \underbrace{\frac{2}{\tau}(\boldsymbol{\sigma} - \mathbf{1})}_{\textit{relaxation}} \end{cases}$$

where...

$\sigma_{ij} \equiv \frac{\langle R_i R_j \rangle}{R_0^2}$	polymer conformation tensor
$\boldsymbol{R} = (R_1, ..., R_d)$	polymer end-to-end separation
R_0	radius of gyration

Viscoelastic Kolmogorov flow (1)

Let us consider a $2D$ parallel flow

$$\boldsymbol{f} = (F\cos(y/L), 0) \qquad \text{Kolmogorov forcing}$$

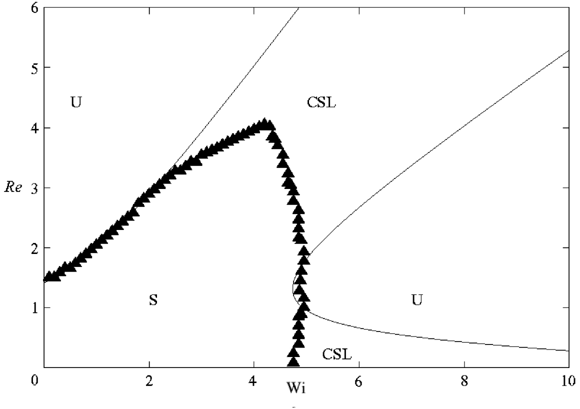
Laminar fixed point:

$$\boldsymbol{u} = (U_0 \cos(y/L), 0)$$

$$\boldsymbol{\sigma} = \begin{pmatrix} 1 + \tau^2 \frac{U_0^2}{2L^2} \sin^2(y/L) & -\tau \frac{U_0}{2L} \sin(y/L) \\ -\tau \frac{U_0}{2L} \sin(y/L) & 1 \end{pmatrix}$$

$$F = [\nu U_0 (1+\eta)]/L^2; \quad Wi \equiv \frac{U_0 \tau}{L}; \quad Re \equiv \frac{U_0 L}{\nu(1+\eta)}$$

Viscoelastic Kolmogorov flow (2)



G. Boffetta, A. Celani, A. Mazzino, A. Puliafito, M. Vergassola, J. Fluid Mech. **523**, 161 (2005)

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Flow resistance and momentum budget

$P = \langle \boldsymbol{f} \cdot \boldsymbol{u} \rangle$ power injection

$P_{lam} = \frac{U_0^2 \nu (1+\eta)}{2L^2}$; $\langle u_x \rangle = U \cos(y/L)$, $\langle \sigma_{xy} \rangle = -\Sigma \sin(y/L)$ for KF

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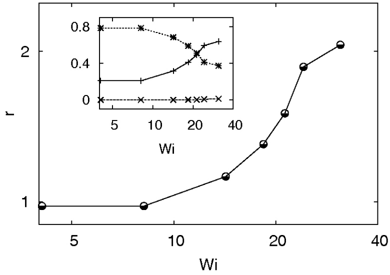
$$r \equiv \frac{P}{P_{lam}} = \frac{FL^2}{\nu(1+\eta)U}$$



$E\ell \equiv \frac{Wi}{Re} = 64;\ Wi = \tau U/L;\ Re < 0.48$

$\tau = 4;\ L = 1/4;\ \nu(1+\eta) = 1$

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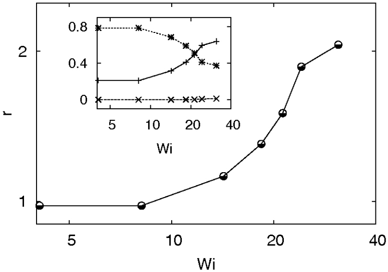

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$\tau = 4$; $L = 1/4$; $\nu(1+\eta) = 1$

$$\partial_y \Pi_R = \partial_y(\Pi_\nu + \Pi_P) + f_x$$

$\Pi_P \equiv \frac{2\nu\eta}{\tau}\langle \sigma_{xy} \rangle$; $\Pi_\nu \equiv \nu \partial_y \langle u_x \rangle$

$\Pi_R \equiv \langle u_x u_y \rangle = U_2 \sin(y/L)$ (numerical obs.)

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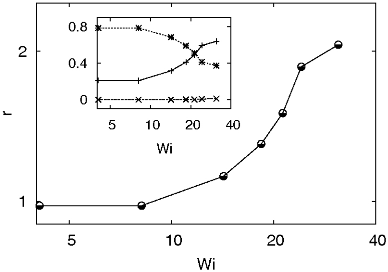

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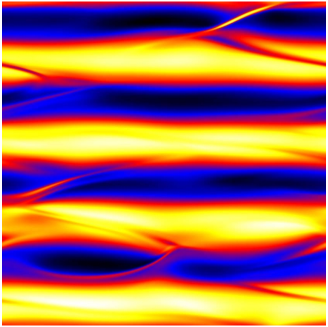
$$FL = U_2 + \frac{\nu U}{L} + \frac{2\nu\eta}{\tau}\Sigma$$

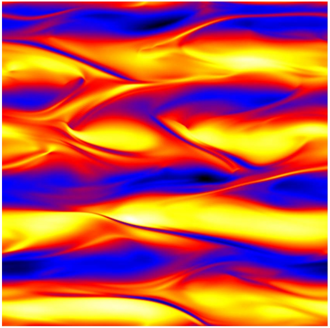
$\frac{U_2}{FL} < 10^{-2} \implies$ the “turbulent” stress is due to elasticity

Chaotic flow (1)

Snapshots of the vorticity field

$Wi = 21.3$ $\qquad$ $Wi = 31$



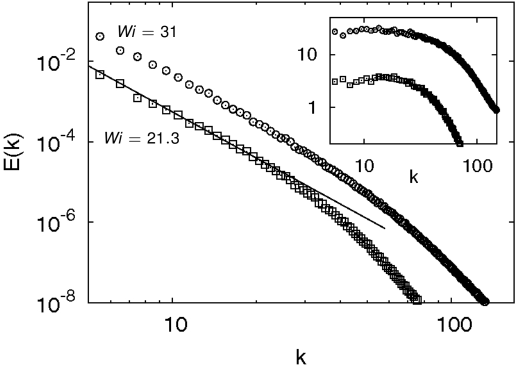


- ○ $\omega_{max} = U/L$
- ● $\omega_{min} = -U/L$

The flow develops active modes at all the scales

Chaotic flow (2)

Spectra of velocity fluctuations
in the turbulent-like states

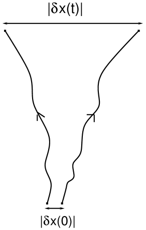


$E(k) \sim k^{-\alpha}$

$3 < \alpha < 4$

Elastic turbulence corresponds to a smooth chaotic flow

Mixing: Lagrangian Lyapunov exponent



$$\lambda_L \equiv \lim_{t\to\infty} \lim_{|\delta \boldsymbol{x}(0)|\to 0} \frac{1}{t} \ln \frac{|\delta \boldsymbol{x}(t)|}{|\delta \boldsymbol{x}(0)|}$$

$$G(\gamma) : \; P_t(\gamma) \sim e^{-t\,G(\gamma)}$$

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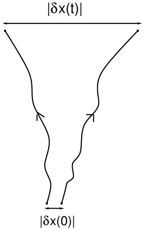

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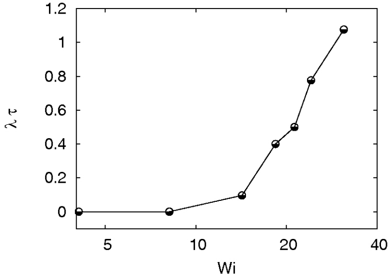
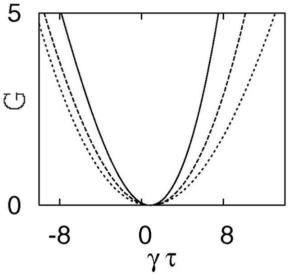
Lyapunov exponent



Cramer function



$Wi = 18.4$

$Wi = 24.2$

$Wi = 31$

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Conclusions

The basic phenomenology of elastic turbulence is reproduced in $2D$.
For $Wi > Wi_c \sim 10$:

1. the flow resistance grows with respect to the laminar case;
2. the flow displays features of a strongly non-linear state;
3. the Lagrangian Lyapunov exponent is positive and grows with Wi; the distribution of its fluctuations becomes asymmetric.

KF in the lab

Uspekhi Mat. Nauk 38:4 (1983), 101–111 Russian Math. Surveys 38:4 (1983), 113–126

Kolmogorov flow and laboratory simulation of it

A.M. Obukhov

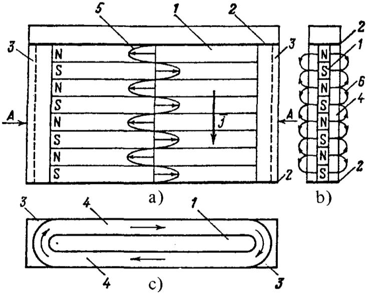

Fig.2. A vertical MHD arrangement for simulating a Kolmogorov flow. Front view, side view, and top view (cut along A–A). 1) a sheet of magnetoelastic rubber of size $245 \times 180 \times 5$ mm^3; 2) copper electrode; 3) side wall rounded on the inside; 4) the channel of flow for the fluid; 5) profile of the MHD force; 6) lines of force of the magnetic field.

Experiment by A.M. Batchaev, V.A. Dovzhenko

Cholesky decomposition

Numerical integration:

$2D$ Oldroyd-B $\Rightarrow \omega,\ \sigma_{11},\ \sigma_{22},\ \sigma_{21} = \sigma_{12}$

$\boldsymbol{\sigma}$ is positive definite $\Longrightarrow \boldsymbol{\sigma} = \boldsymbol{L}\boldsymbol{L}^T$

$\boldsymbol{L}$ is a lower triangular matrix:

$$\ell_{11} = \sqrt{\sigma_{11}}$$
$$\ell_{21} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} = \frac{\sigma_{12}}{\ell_{11}}$$
$$\ell_{22} = \sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}} = \frac{\det \boldsymbol{\sigma}}{\ell_{11}}$$

T. Vaithianathan, L. R. Collins, J. Comp. Physics **187**, 1 (2003)

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$$\boxed{\tilde{\ell}_{ij} \longrightarrow \ell_{ii} = e^{\tilde{\ell}_{ii}};\ \ell_{ij} = \tilde{\ell}_{ij} \longrightarrow \boldsymbol{\sigma} = \boldsymbol{L}\boldsymbol{L}^T}$$

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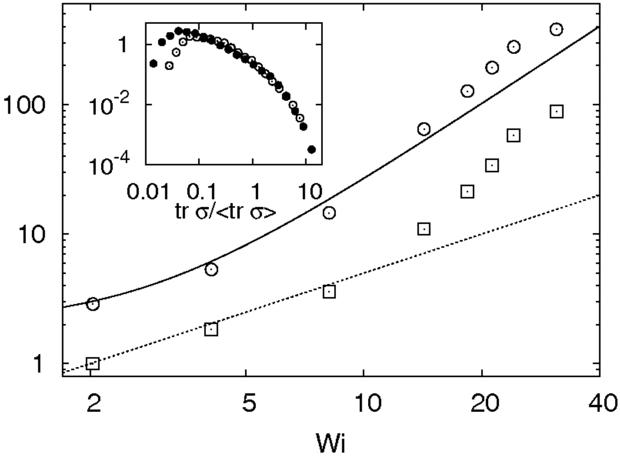
Polymer statistics

laminar behaviours:



$$\langle \mathrm{tr}\boldsymbol{\sigma} \rangle = 2 + \frac{Wi^2}{4}$$

$$\Sigma = \frac{Wi}{2}$$

The PDF of $\mathrm{tr}\boldsymbol{\sigma}$ shows elongations up to $15\langle \mathrm{tr}\boldsymbol{\sigma} \rangle$ with a distribution that, for strong elongations, becomes independent of *Wi*.