• EX. 1

$\frac{V_1}{V_2} = \nu = 3$ COMPRESSION

(A) $T = \text{const}$ (ISOTHERME)

Q1) $P_1 V_1 = m r T_1$
$P_2 V_2 = m r T_2$ (G.P.) ; $T_1 = T_2 \Rightarrow P_1 V_1 = P_2 V_2 \Rightarrow$

$\Rightarrow \frac{P_2}{P_1} = \frac{V_1}{V_2} = \nu \Rightarrow$

$\Rightarrow P_2 = \nu P_1 = 3 P_1 > P_1$

Q2) $W_{12} = -\int_{V_1}^{V_2} p\,dV = -mrT \ln\left(\frac{V_2}{V_1}\right) = mrT \ln \nu$

($T_1 = T_2 = T$)

$\nu > 1 \Rightarrow \ln \nu > 0 \Rightarrow W_{12} > 0$ (reçu, en accord avec : compression)

Q3) $\Delta U_{12} = W_{12} + Q_{12}$ (1° PRINCIPE)

$T_1 = T_2 \Rightarrow \Delta U = m c_v (T_2 - T_1) = 0 \Rightarrow$ (G.P.)

$\Rightarrow Q_{12} = -W_{12} < 0$ (cédée)

(B) $\delta Q = 0$ (ADIABATIQUE)

Q4) REVERSIBLE $\Rightarrow P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ (Loi de LAPLACE)

avec $\gamma = \frac{c_p}{c_v} = \frac{c_v + r}{c_v} > 1$

$\Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = \nu^{\gamma} P_1 > P_1$
$= 3^{\gamma} P_1$

Q5) $T_2 = \frac{P_2 V_2}{mr} = \frac{P_1 \nu^{\gamma} V_2}{mr} = \frac{P_1 V_1}{mr} \nu^{\gamma - 1} = \nu^{\gamma - 1} T_1 > T_1$

(G.P. ; $P_2 = \nu^{\gamma} P_1$; $V_2 = \frac{V_1}{\nu}$; T_1)

Q6) $\Delta U_{12} = W_{12} + Q_{12}$

$Q_{12} = 0 \Rightarrow W_{12} = \Delta U_{12} = m c_v (T_2 - T_1)$ (↑ G.P.)

$T_2 > T_1 \Rightarrow W_{12} > 0$ (reçu).

Q7) $\Delta S_{TOT} = \Delta S_{GAZ} + \Delta S_{TH.}$

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THERMOSTAT à $T = T_e$

(B) $\delta Q = 0 \Rightarrow \Delta S_{GAZ} = \Delta S_{TH.} = \Delta S_{TOT} = 0$

(A) $\Delta S_{GAZ} = \frac{Q_{12}}{T}$ (↑ $T = const.$) ; $\Delta S_{TH.} = -\frac{Q_{12}}{T_e}$ $\Rightarrow$

$\Rightarrow \Delta S_{TOT} = Q_{12} \left(\frac{1}{T} - \frac{1}{T_e} \right) = Q_{12} \frac{T_e - T}{T T_e}$

(i) $T < T_e \Rightarrow Q_{12} > 0$; $T_e - T > 0 \Rightarrow \Delta S_{TOT} > 0$

(ii) $T > T_e \Rightarrow Q_{12} < 0$; $T_e - T < 0 \Rightarrow \Delta S_{TOT} > 0$

• Ex. 2

$m = 1\,kg$

$p_1 = 1\,atm = 10^5\,Pa$

$p_2 = 15\,p_1$

$V_1 = 0{,}83\,m^3$

$V_3 = 0{,}27\,m^3$

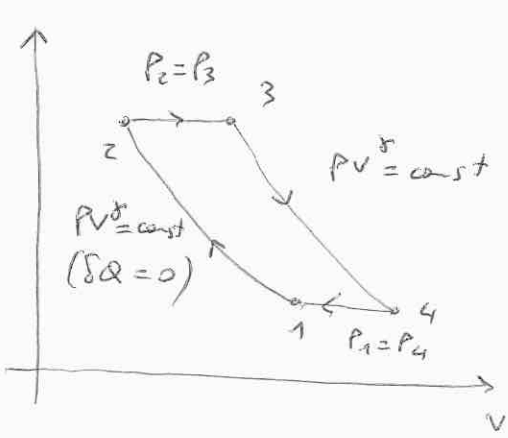
$c_v = 713\,\frac{J}{kg\,K}$

$r = 287\,\frac{J}{kg\,K}$

$c_p = c_v + r$

$\gamma = \frac{c_p}{c_v} \simeq 1{,}4$

Q1)



Q2) $T_1 = \frac{p_1 V_1}{m r} \simeq 289\,K$

Q3) $p_1 V_1^\gamma = p_2 V_2^\gamma$ (Loi de LAPLACE)

$V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} V_1 \simeq 0{,}12\,m^3$

$T_2 = \frac{p_2 V_2}{m r} \simeq 627\,K$

Q4) $Q_{12} = 0 \Rightarrow W_{12} = \Delta U_{12} = m c_v (T_2 - T_1) \simeq 240984\,J$

$W_{12} > 0$ (reçu)

Q5) $p_3 = p_2 \Rightarrow T_3 = \frac{p_3 V_3}{m r} = \frac{p_2 V_3}{m r} \simeq 1411\,K$

Q6) $W_{23} = -p_2 (V_3 - V_2) = -225000\,J < 0$ (cédé)

$p = const = p_2$

$\Delta U_{23} = m c_v (T_3 - T_2) = 558992\,J$

$Q_{23} = \Delta U_{23} - W_{23} = 783992\,J > 0$ (reçue)

Q7)

$$P_4 = P_1 = 10^5 \text{ Pa}$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \Rightarrow V_4 = \left(\frac{P_3}{P_4}\right)^{\frac{1}{\gamma}} V_3 = \left(\frac{P_3}{P_1}\right)^{\frac{1}{\gamma}} V_3 =$$

($P_4 = P_1$)

$$= \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} V_3 \simeq 1{,}87 \text{ m}^3$$

($P_3 = P_2$)

$$T_4 = \frac{P_4 V_4}{nR} \simeq 652 \text{ K}$$

Q8) $Q_{34} = 0 \Rightarrow W_{34} = \Delta U_{34} = n c_v (T_4 - T_3) = -54167 \text{ J} < 0$ (cédé)

Q9) $W_{41} = -P_1 (V_1 - V_4) = 104000 \text{ J} > 0$ (reçu)

($p = \text{const}$)

$$\Delta U_{41} = n c_v (T_1 - T_4) = -258819 \text{ J}$$

$$Q_{41} = \Delta U_{41} - W_{41} = -362819 \text{ J} < 0 \quad \text{(cédée)}$$

Q10) $Q_{in} = Q_{23} = 783992 \text{ J} > 0$

$$Q_{out} = Q_{41} = -362819 \text{ J} < 0$$

$$\eta = \frac{|W|}{Q_{in}} = 1 + \frac{Q_{out}}{Q_{in}} \simeq 0{,}54$$

$\Delta U = 0$ (cycle) $\Rightarrow W = -(Q_{in} + Q_{out})$

$W < 0 \Rightarrow |W| = -W$

Q11)

$$\eta = 1 + \frac{Q_{out}}{Q_{in}} = 1 + \frac{Q_{41}}{Q_{23}}$$

$p = const \Rightarrow$ $Q_{41} = \Delta H_{41} = m c_p (T_1 - T_4)$ $\Rightarrow$

$Q_{23} = \Delta H_{23} = m c_p (T_3 - T_2)$

$$\Rightarrow \eta = 1 + \frac{T_1 - T_4}{T_3 - T_2}$$

Q12) $$\eta = 1 + \frac{T_1 - T_4}{T_3 - T_2} = 1 - \frac{T_1}{T_2} \cdot \frac{T_4/T_1 - 1}{T_3/T_2 - 1}$$

$P_1 = P_4$ $\quad P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$P_2 = P_3$ $\quad P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$

$$\Rightarrow \frac{T_4}{T_1} = \frac{P_4 V_4}{P_1 V_1} = \frac{V_4}{V_1}$$

$$\frac{T_3}{T_2} = \frac{P_3 V_3}{P_2 V_2} = \frac{V_3}{V_2}$$

$$\frac{P_3 V_3^{\gamma}}{P_2 V_2^{\gamma}} = \frac{P_4 V_4^{\gamma}}{P_1 V_1^{\gamma}} \Rightarrow \left(\frac{V_3}{V_2}\right)^{\gamma} = \left(\frac{V_4}{V_1}\right)^{\gamma} \Rightarrow \frac{V_3}{V_2} = \frac{V_4}{V_1}$$

$$\Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2} \Rightarrow \eta = 1 - \frac{T_1}{T_2}$$

RAPPORT des TEMPÉRATURE $T_{12} \equiv \frac{T_2}{T_1} \Rightarrow \boxed{\eta = 1 - \frac{1}{T_{12}}}$

Q13) $$T_{12} = \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} = \frac{P_2^{1-\frac{1}{\gamma}} P_1^{\frac{1}{\gamma}}}{P_1} = \frac{P_2^{\frac{\gamma-1}{\gamma}}}{P_1^{\frac{\gamma-1}{\gamma}}} =$$

$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} V_1$

$P_{12} \equiv \frac{P_2}{P_1}$ RAPPORT des PRESSIONS

$$= \left(P_{12}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \boxed{\eta = 1 - \frac{1}{P_{12}^{\frac{\gamma-1}{\gamma}}}}$$