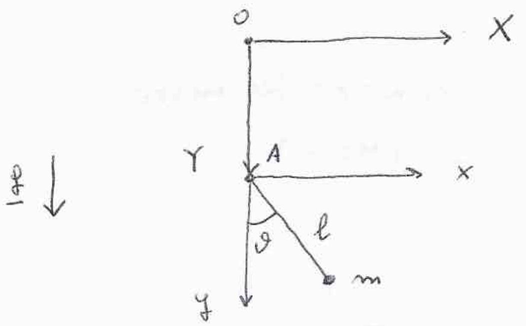
TD3

- EX. 1 :



$\underline{OA} = h(t)\,\hat{Y}$

Q1) Axy : GALILÉEN (préliminairement) → p.ex. : Axy FIXE

$L = T - V$

$T = \frac{m}{2} v^2$; $v = \ell \dot{\vartheta}$ => $T = \frac{m}{2} \ell^2 \dot{\vartheta}^2$

$V = - mg\ell\cos\vartheta$
↑
$\underline{f} = m\underline{g}$ FORCE APPLIQUÉE ; $\underline{f} = mg\,\hat{y}$; $f_y = -\frac{\partial V}{\partial y}$

$V = -mgy = -mg\ell\cos\vartheta$

$$\Rightarrow \boxed{L = T - V = \frac{m}{2}\ell^2\dot{\vartheta}^2 + mg\ell\cos\vartheta}$$

Q2) EQ. LAGRANGE

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j \iff \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

$$\Rightarrow \frac{d}{dt}\underbrace{\left(\underbrace{\frac{\partial L}{\partial \dot{\vartheta}}}_{m\ell^2\dot{\vartheta}}\right)}_{m\ell^2\ddot{\vartheta}} - \underbrace{\frac{\partial L}{\partial \vartheta}}_{-mg\ell\sin\vartheta} = 0 \Rightarrow \cancel{m}\ell^{\cancel{2}}\ddot{\vartheta} + \cancel{m}g\cancel{\ell}\sin\vartheta = 0 \Rightarrow$$

$$\Rightarrow \boxed{\ddot{\vartheta} + \frac{g}{\ell}\sin\vartheta = 0}$$

Q3) MAINTENANT Axy en MOUVEMENT

$\underline{OA} = h(t)\,\hat{Y}$ VECTEUR POSITION de l'ORIGINE du RÉF. en MOUVEMENT

$\underline{x} = (x, y)$ dans Axy

$\underline{R} = \frac{1}{m}\, m\, \underline{x} = \left(l \sin\vartheta, l\cos\vartheta\right)$ CENTRE de MASSE ($M \equiv m$)

$$\underline{a}^{(e)} = \frac{d^2}{dt^2}\underline{OA} = \ddot{h}\,\hat{Y}$$

$$Q_j^{(e)} \equiv Q_\vartheta^{(e)} = -m\,\underbrace{\underline{a}^{(e)}}_{\ddot{h}\hat{Y} = \ddot{h}\hat{y}} \cdot \underbrace{\frac{\partial \underline{R}}{\partial \vartheta}}_{l\cos\vartheta\,\hat{x} - l\sin\vartheta\,\hat{y}} = m l \ddot{h} \sin\vartheta$$

$$Q_j^{(COR)} = Q_j^{(CENT)} = 0$$

Q4) EQ. LAGRANGE

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j + Q_j^{(e)} \quad ; \quad q_\vartheta \equiv \vartheta$$

$$Q_\vartheta = -\frac{\partial V}{\partial \vartheta} = -mgl\sin\vartheta$$

$$V = -mgl\cos\vartheta$$

$$Q_\vartheta^{(e)} = ml\ddot{h}\sin\vartheta$$

$$T = \frac{m}{2} l^2 \dot{\vartheta}^2$$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\vartheta}}\right)}_{ml^2\ddot{\vartheta}} - \underbrace{\frac{\partial T}{\partial \vartheta}}_{0} = Q_\vartheta + Q_\vartheta^{(e)} \Rightarrow \cancel{m}l^{\cancel{2}}\ddot{\vartheta} = -\cancel{m}g\cancel{l}\sin\vartheta + \cancel{ml}\ddot{h}\sin\vartheta$$

$$\Rightarrow \boxed{\ddot{\vartheta} + \frac{g - \ddot{h}}{l}\sin\vartheta = 0}$$

Q5) EQUILIBRE

$$\ddot{\vartheta} + \frac{g - \ddot{h}}{\ell} \sin\vartheta = 0 \quad ; \quad \text{EQUILIBRE : } \ddot{\vartheta} = -\frac{g-\ddot{h}}{\ell}\sin\vartheta = 0$$

$\ddot{h} = \text{cost}$ par hypothèse

(i) $\ddot{h} = 0$ (c'est-à-dire $\dot{h} = \text{cost}$) $\Rightarrow$ Axy INERTIEL

$\Rightarrow \sin\vartheta = 0 \Rightarrow \vartheta = 0, \pi$

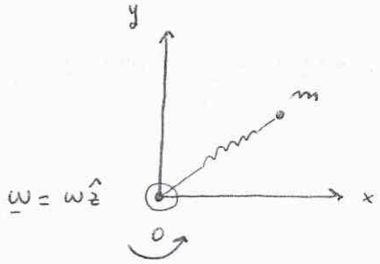
(ii) $0 < \ddot{h} < g \Rightarrow \sin\vartheta = 0 \Rightarrow \vartheta = 0, \pi$

$$\frac{g - \ddot{h}}{\ell} > 0$$

(iii) $\ddot{h} = g \Rightarrow g - \ddot{h} = 0 \Rightarrow \forall\vartheta$

Dans le référentiel Axy : $\ddot{\vartheta} = 0$ (SYSTÈME LIBRE)

• EX. 2 :



O_{xy} en ROTATION UNIFORME : $\underline{\omega} = \omega \hat{z}$; $\dot{\omega} = 0$

Q1)

$$\text{MASSE } m \begin{cases} \text{POSITION} : \underline{r} = (x, y) \\ \text{VITESSE} : \underline{v} = (\dot{x}, \dot{y}) \end{cases}$$

$$T = \frac{m}{2} v^2 = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right)$$ EN. CINETIQUE

$$\underline{F} = -k\underline{r} = -k(x, y) = \left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}\right)$$

$$V = \frac{k}{2} r^2 = \frac{k}{2}\left(x^2 + y^2\right)$$ EN. POTENTIELLE

$$Q_x = -\frac{\partial V}{\partial x} = -kx \; ; \; Q_y = -\frac{\partial V}{\partial y} = -ky$$ FORCES (ELASTIQUES)

Q2) FORCES INERTIELLES

$q_1 \equiv x$; $q_2 \equiv y$

ENTRAINEMENT: $Q_j^{(e)} = 0$ (O : FIXE)

$$\text{CENTRIFUGE} : \begin{cases} Q_x^{(\text{CENT})} = \dfrac{\partial}{\partial x}\left[\dfrac{m}{2}\left(\underline{\omega} \times \underline{r}\right)^2\right] = m\omega^2 x \\ Q_y^{(\text{CENT})} = \dfrac{\partial}{\partial y}\left[\dfrac{m}{2}\left(\underline{\omega} \times \underline{r}\right)^2\right] = m\omega^2 y \end{cases}$$

$$\underline{\omega} \times \underline{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix} = \hat{x}(-\omega y) + \hat{y}\,\omega x \Rightarrow \left|\underline{\omega} \times \underline{r}\right|^2 = \omega^2\left(x^2 + y^2\right)$$

CORIOLIS : $\begin{cases} Q_x^{(COR)} = \dfrac{\partial}{\partial x}(\underline{\omega}\cdot\underline{L}) - \dfrac{d}{dt}\left(\dfrac{\partial(\underline{\omega}\cdot\underline{L})}{\partial\dot{x}}\right) \\ Q_y^{(COR)} = \dfrac{\partial}{\partial y}(\underline{\omega}\cdot\underline{L}) - \dfrac{d}{dt}\left(\dfrac{\partial(\underline{\omega}\cdot\underline{L})}{\partial\dot{y}}\right) \end{cases}$

MOMENT ANGULAIRE: $\underline{L} = m\,\underline{r}\times\underline{v} = m\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & 0 \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = \hat{z}\,(x\dot{y} - y\dot{x})\,m$

$\Rightarrow \quad \underline{\omega}\cdot\underline{L} = \omega\,\hat{z}\cdot m(x\dot{y} - y\dot{x})\,\hat{z} = m\,\omega\,(x\dot{y} - y\dot{x}) \quad \Rightarrow$

$\Rightarrow$ CORIOLIS: $\begin{cases} Q_x^{(COR)} = \dfrac{\partial}{\partial x}\left[m\omega(x\dot{y} - y\dot{x})\right] - \dfrac{d}{dt}\left[\dfrac{\partial}{\partial\dot{x}}\left(m\omega(x\dot{y} - y\dot{x})\right)\right] = \\ \qquad = m\omega\,\dot{y} + \dfrac{d}{dt}\left(m\omega y\right) = 2m\omega\,\dot{y} \\ Q_y^{(COR)} = \dfrac{\partial}{\partial y}\left[m\omega(x\dot{y} - y\dot{x})\right] - \dfrac{d}{dt}\left[\dfrac{\partial}{\partial\dot{y}}\left(m\omega(x\dot{y} - y\dot{x})\right)\right] = \\ \qquad = -m\omega\,\dot{x} - \dfrac{d}{dt}\left(m\omega x\right) = -2m\omega\,\dot{x} \end{cases}$

Q3) EQ. LAGRANGE

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j + \underbrace{\cancel{Q_j^{(R)}}}_{=0} + Q_j^{(CENT)} + Q_j^{(COR)} \quad \Rightarrow$$

$\left[T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)\ ;\ Q_x = -kx\ ;\ Q_y = -ky\right]$

$$\Rightarrow \begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{x}}\right) - \underbrace{\cancel{\dfrac{\partial T}{\partial x}}}_{=0} = Q_x + Q_x^{(CENT)} + Q_x^{(COR)} & (x) \\ \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{y}}\right) - \underbrace{\cancel{\dfrac{\partial T}{\partial y}}}_{=0} = Q_y + Q_y^{(CENT)} + Q_y^{(COR)} & (y) \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{\begin{cases} m\ddot{x} = -kx + m\omega^2 x + 2m\omega\dot{y} \\ m\ddot{y} = -ky + m\omega^2 y - 2m\omega\dot{x} \end{cases}}$$

EQUATIONS de LAGRANGE